

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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NOVEMBER 8, 1912.

No. 1

F. W. DYSON, Esq., M.A., LL.D., F.R.S., PRESIDENT, in the Chair.

Alexander Granville Blake, Rossall School, Fleetwood, Lancashire;

Arthur Lock, Navigating Officer, s.s. *Rohilla*, 268 Newport Road, Cardiff;

Algernon Montagu Newbegin, Inverurie, Talbot Road, Carshalton, Surrey; and

Thomas Nathaniel Orchard, Doctor of Medicine, Ashfield House, Seedley Road, Pendleton, Manchester,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Lieut. William Gordon Bennett, R.N.R., Endlestone, Battenhall Road, Worcester (proposed by P. Groves-Showell);

R. T. Crawford, Ph.D., Associate Professor of Practical Astronomy in the Berkeley Astronomical Department of the University of California, Berkeley, Cal., U.S.A. (proposed by A. O. Leuschner);

H. B. Curlewis, B.A., Acting Government Astronomer for Western Australia, The Observatory, Perth, Western Australia (proposed by W. Ernest Cooke);

Rev. Ben Dudley, Bluff Harbour, New Zealand (proposed by the Rev. Daniel Dutton);

- Rev. James Molesworth Franklin Dumphreys, L.Th., 5 Donaldson Road, Kilburn, N.W. (proposed by J. R. Leeson);
- Benjamin Gregory, Grasmere, Glencairn Park Road, Cheltenham (proposed by J. M'Carthy);
- Edward Hawkins, Teacher of Mathematics and Navigation in the Municipal College, Portsmouth (proposed by Thomas Lewis);
- Captain Frederick John Horne, L.C.C. School of Navigation, High Street, Poplar; E. (proposed by P. Groves-Showell);
- John Jackson, B.A., Trinity College, Cambridge (proposed by F. J. M. Stratton);
- William C. Jenkins, Curator, Godlee Observatory, Manchester, 11 Upper Lloyd Street, Moss Side, Manchester (proposed by E. T. Whitelow);
- John Hall Rider, M.Inst.C.E., P.O. Box 149, Johannesburg, South Africa (proposed by R. T. A. Innes);
- Robert Rossi, M.Sc., Research Student, Trinity College, Cambridge (proposed by H. F. Newall);
- Thomas Royds, D.Sc., Assistant Director, Kodaikānal Observatory, India (proposed by John Evershed);
- William James Simmons, Solicitor, 8/2 Hastings Street, Calcutta, India (proposed by H. G. Tomkins);
- Oliver Rowland Walkey, 11 Grange Road, Chiswick, W. (proposed by L. N. G. Filon);
- Thomas Petty Witter, Ayrefield House, Appley Bridge, near Wigan, Lancashire (proposed by F. W. Longbottom); and
- Harry Thomas Wright, A.R.C.Sc. (Lond.), Engineering College, Tāng Shan, near Tientsin, N. China (proposed by A. H. Stuart).

One hundred and fifty-five presents were announced as having been received since the last meeting, including, amongst others:—

Portrait of John Goodricke, presented by Mr. C. A. Goodricke; A. Auwers, Bradley's *Beobachtungen an alten Instrumenten*, Bd. I., presented by the author; Royal Society, Record of the Society, and facsimile of Signature Book, presented by the Society; Cordoba Observatory, Southern Celestial Charts, new edition, charts 1-3, presented by the Observatory; Helwān Observatory, Photographs of Nebulæ by H. Knox Shaw (13 slides), presented by the Observatory; A series of Transparencies of Halley's Comet made at the Union Observatory, Johannesburg, presented by the Observatory.

Astrographic Chart: 20 charts from the San Fernando Observatory; 38 charts from the Tacubaya Observatory, and 10 charts from the Observatory, Uccle, Belgium, presented by the Observatories.

Gift to the Society of a Portrait of John Goodricke.

The Society has received from Mr. C. A. Goodricke the gift of a pastel portrait of John Goodricke, of York. Accompanying the gift was the following letter, which was read at the meeting on 1912 Nov. 8, when the gift was announced, and the thanks of the Society were given to Mr. Goodricke. In accordance with the wish of the meeting, the Secretaries have the pleasure of publishing the letter to the Society.

27 CREDITON ROAD,
WEST HAMPESTEAD, LONDON, N.W.
November 8th, 1912.

The Secretary,
Royal Astronomical Society,
Burlington House.

Dear Sir,—As you are aware, I bequeathed to the Royal Astronomical Society my original (pastel) portrait of Mr. John Goodricke, but on further consideration I have decided, as I intimated to you in the summer, that I would prefer the Society should possess it at once.

I have therefore much pleasure in now, personally, handing the picture over to you, and I desire to say that I fully appreciate their acceptance of it, and that in according to it an honourable position in their rooms, I realise the tribute the Council are paying to the memory of one who I understand was worthy of the distinction he obtained by his zeal and by his discoveries in the most fascinating of all the sciences.

Mr. Goodricke's work is too well known to the Society to need any mention by me, but the present opportunity is, perhaps, a fitting and convenient one for me to state a few facts regarding the subject of the picture and the picture itself, which it may be thought desirable the Society should know.

Mr. John Goodricke was the elder son of Mr. Henry Goodricke, of York, and was at the time this portrait was executed (1785) heir to his grandfather, the Right Honourable Sir John Goodricke, Baronet, of Ribston and Bramham Park, both in the county of York, in consequence of his father's death in 1784.

John Goodricke was born at Groningen, 17th Sept. 1764, and baptized two days later in the Anglican Church there. He died, unmarried, at York, on the 20th April 1786, aged twenty-two years only, and was buried in the Goodricke family vault at Hunsingore Church.

He has been described as a very remarkable person, and when his ability is remembered, and, above all, the fact that he was deaf and dumb from birth, this description of him will readily be granted. He was placed as a pupil at the academy at Warrington, of which the Rev. William Turner was head master, and here he made good progress in classics and mathematics.

The value and importance of Mr. Goodricke's discoveries obtained for him, in 1783, from the Royal Society, their highest honour—the Sir Godfrey Copley Medal.

It may be of some interest to know that the observatory at York in which Mr. Goodricke worked, together with Mr. Edward Pigott, was a small building or summer-house in the garden behind Mr. Pigott's house in Bootham, about 400 yards north-west of the Minster.

Mr. Goodricke lived in a house in Lendal, York, with his father and mother. Here his father died 9th July 1784. John Goodricke also died suddenly in this house on 20th April 1786, while he was in the midst of his investigations, his untimely end having, it is said, been occasioned by exposure to cold during his astronomical observations. His death occurred only a fortnight after his election to the Royal Society.

As regards the portrait. It was a late addition to a large collection of post-Reformation family portraits at Ribston, Yorkshire, the seat of the Goodricke family since 1542. When the Goodricke estates were sold in 1836 all the portraits were removed, and that of John Goodricke, the astronomer, found a home with a few others with Mrs. Fairfax, at Gilling Castle, co. York. Mrs. Fairfax was a sister of the astronomer. Not long after the death of Mrs. Barnes, who was Mrs. Fairfax's daughter, and who possessed Gilling for her life, that place was sold, and the Goodricke portraits which were there were offered to me by the purchaser of that property in 1898.

In this manner, then, this portrait of the Astronomer, which has hung successively in his own home in Lendal, York, in the Goodricke family mansion, Ribston Hall, and in the ancient home of the Fairfaxes, at Gilling Castle, became my property.

It affords me no small degree of satisfaction to know that this picture has now found a permanent home, freed from the possible vicissitudes which so frequently beset private property, and that it will henceforth be valued and preserved with care by the Royal Astronomical Society.

In conclusion, I may mention that this portrait is considered by artists as a very fine work, and it has been ascribed by Mr. Algernon Graves, the art expert, to James Scouler, a portrait painter who exhibited at the Royal Academy between the years 1769 and 1787.—I am, yours faithfully, C. A. GOODRICKE.

Spectrographic Observations of the Sun's Rotation at Cambridge Observatory. By J. B. Hubrecht, M.A.

(Communicated by Professor Newall.)

The Sun's rotation was first quantitatively investigated with the spectroscope by Dunér, whose visual observations, made at Lund, were published in his celebrated memoir *Recherches sur la Rotation du Soleil* of 1891. After a lapse of time the subject was again taken up visually in 1898 by the same author* and by Halm in 1901 at Edinburgh,† while Adams at Mount Wilson has in a series of papers, mostly published in the *Astrophysical Journal*,‡ dealt extensively with his photographic investigations of the solar rotation which he began in 1906, as was also done by Messrs. Storey and Wilson with their photographic continuation (1908–1910) of Halm's work, in a paper communicated to the Society by the Astronomer Royal for Scotland.§ Recently the scope of the work has been extended on a co-operative basis between several observatories by decisions taken at the meeting of the International Solar Union at Mount Wilson in 1910.

The present paper deals with a part of the contribution assigned to the Cambridge Observatory in this co-operative scheme. The research had been started as far back as January 1907, when the new spectrograph arrived at Cambridge Observatory, but as the necessary preliminary investigations took considerable time, it was possible to bring the actual observations into line with the general programme as adopted by the Solar Union in 1910.

The horizontal equipment used at Cambridge for obtaining a non-rotating image on the slit plate of the spectrograph consists of a cœlostat mirror of 16 inches [40 cm.] diameter, an equally large secondary mirror reflecting the beam from the cœlostat mirror horizontally northwards, and an achromatic lens of 12 inches [30 cm.] diameter and 60 foot [18 m.] focus in this beam. These are all mounted in a little building, provided with the necessary shutters, on the south side of the Newall dome. The light travels from the lens through an opening in the south wall of the dome nearly diametrically through it and out at the other end into a room on the north side of the dome, the "North Room," where the spectrograph is installed. In the dome the beam is entirely unenclosed, an arrangement which has proved very satisfactory in giving excellent definition of the image whenever atmospheric conditions were favourable. The image lens itself, made by Messrs. Cooke & Sons of York, though sufficiently good for this research, is only a temporary one, as it has been found to

* *Nova Acta, Roy. Soc. Scient. Upsal.* Series 4, I., No. 6, 1906.

† *Astr. Nachr.*, clxxiii. 287, 1907.

‡ *Astrophysical Journal*, xxvi. 203, 1907; xxvii. 213, 1908; xxix. 110, 1909, etc.

§ *Monthly Notices*, lxxi. 674, 1911.

be not optically perfect enough for the most refined sunspot work. It is mounted on a horizontal sliding carriage and provided with means of movement on this carriage for focussing purposes from the image end of the beam. From the same place it is possible, by means of cords, to work the slow motions of the secondary mirror and thus move the Sun's image in any desired direction across the slit plate of the spectrograph.

The spectrograph is the Littrow instrument described by Professor Newall in *Monthly Notices*, lxviii. p. 6, and was devised by him purposely for sunspot and rotation work. The tube of the spectrograph is a heavy cast-iron tube, 9 inches [23 cm.] in diameter with walls $\frac{1}{4}$ inch [0.6 cm.] thick. To one end of the tube is fitted a casting which carries the slit plate and the photographic plate-holder, the middle point of the slit being separated by 65 mm. from the middle point of the photographic plate. The other end of the tube carries a heavy cast-iron box containing the lens which serves as collimator and camera lens. The lens has an aperture of 4 inches [10 cms.] and a focal length of 14 feet [420 cms.] and is mounted in a focussing tube whose axis can be adjusted to the proper inclination. This heavy box also contains the diffraction grating and a small-angled prism for separating superposed spectra (see below). The tube is supported, with its axis horizontal, on two pairs of rollers, fixed on two brickwork pillars, one pair close to the slit end, the other pair about 2 feet from the other end. The interval between the two bearings is thus 12 feet [360 cms.]. The cylindrical surfaces, which rest on the rollers, are turned on the heavy castings at either end of the tube. In the design of the instrument great attention was paid by Professor Newall to finding the proper position for these bearings, and accordingly it was arranged that the heavy and rigid box containing the lens and grating should project beyond the hind bearing enough to equalise the contrary flexures over that bearing. Experiment shows that the desired equalisation has been very satisfactorily attained. The whole instrument can thus be rotated about its horizontal axis. In this way the slit can be put radial or tangential to the Sun's limb at any desired point. A divided circle 10 inches in diameter is provided for reading off the position angle of the slit.

The grating is a Michelson plane grating, ruled space $4\frac{1}{2}$ by $3\frac{1}{2}$ inches and 15,000 lines per inch. It was found necessary to cover a small strip both at top and bottom in order to eliminate a small aberration which produced a fringe of faint light outside the geometrical edges of the spectrum dependent on the length of slit used. The resolving power is in no way diminished by such a cutting down of the length of the lines only. The centre of the lens in the spectrograph is covered by a diaphragm in order to shut off the light which would otherwise be directly reflected by its nearly plane inner surface and unduly fog the plate.

For the separation of the different orders of spectra a prism of small refracting angle is placed between lens and grating; an

adjustable tilt of the grating attachment enables the observer to get any desired part of the spectrum in the centre of the plate.

When once the slit is in the right position angle for the desired latitude the solar image can be set on the slit by means of lines inscribed on the slit plate. The lines used are shown diagrammatically in fig. 1, where SS' is the slit, TT' a line scribed on the slit plate at right angles to the slit, while, $p_1, p_2 \dots p'_1, p'_2 \dots$ are short lines scribed in such a way that when, as in the figure, the Sun's limb is made to coincide with p_1 and p'_1 the centre s of the slit is a definite amount h inside the point of the Sun's limb where the latter is parallel to the slit. Similarly if, for example, we want observations of another point on the limb in a position 90° removed from the first, but now with the slit radial to the limb, the image

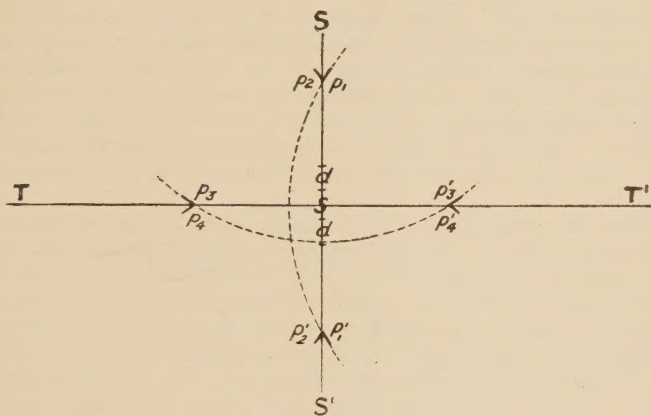


FIG. 1.

has only to be moved so that we get coincidence of the limb, with p_3 and p'_3 . An adjustable diaphragm is placed in front of the slit, used in the present research so that either the central millimeter s was exposed or else the adjoining millimeters d on either side.

An investigation was undertaken, which turned out to be long and troublesome, to ascertain whether rotating the instrument round its axis influenced the position of the spectrum on the plate, and if so to determine accurately the character and amount of this influence. It was found that there appeared in addition to an immediate effect attributed to some outstanding strain in the instrument, a time effect slowly subsiding if the spectrograph was left untouched for some time after turning. The causes of these effects had to be traced and disentangled and, if possible, done away with. Temperature influence appeared to be the principal factor in the time effect; it was very largely reduced by

a very thorough covering up of the whole instrument. Eventually it was decided that no plates were to be taken within half an hour of rotating the tube through 30° , or within a longer period, if it had been necessary to rotate the instrument through larger angles.

These precautions were all-important for the present research, as the exposures on the east and west limb of the Sun were not taken simultaneously. To eliminate any outstanding effect, the procedure in taking the plates was the following:—Half an exposure, say 30 seconds, was given with the east limb on the central part *s* of the slit; then the diaphragm was adjusted, and, with the solar image moved across, a full exposure of 60 seconds was given to the west limb on the adjoining parts *d* of the slit; after that the east limb was again set on the central part *s* and a second exposure of 30 seconds was given on the top of the first. In this way any outstanding “soak” of the instrument has no effect on the displacement to be measured on the plate, provided the “soak” can be regarded as a linear function of the time in the intervals involved. Actually, on all the plates taken the lines in the divided exposure are as well defined as those in the middle exposure. This may be taken as a final justification of the method, and a sign that the outstanding instrumental displacements are negligible.

After this preliminary work was completed the instrument was regarded as ready for regular observations, and it was decided that (1) a series of plates should be taken throughout the year at the equator; this series would afford material collected at a time of sunspot minimum for detection of variations in the equatorial velocity, as well as of possible differences in behaviour of different lines or different elements, for comparison with similar observations to be made at a later time; and (2) if time and climatic conditions allowed, a series of plates at various latitudes all round the Sun should also be taken.

In the meantime the meeting of the International Solar Union had taken place at Mount Wilson and various regions of the spectrum had, by consent of the delegates, been adjudged for observation to different observatories as follows:—

Region.	Observatory.
$\lambda 3800 - \lambda 4000$	Pulkowa.
$\lambda 4000 - \lambda 4140$	Allegheny.
$\lambda 4300 - \lambda 4500$	Cambridge.
$\lambda 4500 - \lambda 4700$	Mount Wilson.
$\lambda 5100 - \lambda 5300$	Mount Wilson.
$\lambda 5500 - \lambda 5700$	Ottawa.
$\lambda 6250 - \lambda 6350$	Edinburgh.

It was further agreed that every observer should, if possible, secure photographs of the spectral region $\lambda 4220 - \lambda 4280$ for the

measurement of a short list of selected lines, the results to serve as a check upon instrumental or personal errors.

Since the dispersion of the Cambridge spectrograph is 1·13 mm. per Angstrom unit in the fourth order, which was used for this work, and since the effective length of the plate is 70 mm., it is seen that at least four plates have to be taken to cover the whole extent of the group assigned to Cambridge. I have in fact restricted myself to region $\lambda 4300$ – $\lambda 4400$, taken on two plates each time, one with $\lambda 4325$, the other with $\lambda 4375$ in the centre. In accordance with the programme fixed by the Solar Union, a number of plates were also to be taken at $\lambda 4250$, to include the spectral region common to all observers.

On these latter plates the 10 lines had been chosen (by the Committee of the Solar Union) for measurement by all the co-operating observatories, while for $\lambda 4300$ – $\lambda 4400$ I made a choice of 40 lines, 20 lines on each plate. The choice was guided by consideration of the assigned origins and the behaviour of the lines in arc, spark, and sunspot, as well as by experimental tests of suitability for accurate measurement. A table of the lines thus chosen is here given, with the wave-lengths, assigned origins, and intensities according to Rowland:—

TABLE I.

Selected Lines in Cambridge Region.

1	4299·149	Ca	3	21	4351·000	Ti	1
2	4300·732	Ti	2	22	4352·083	Mg	5
3	4302·085	Ti	2	23	4352·908	Fe	4
4	4302·692	Ca	4	24	4358·670	Fe	2
5	4307·907	Ca	3	25	4366·061	Fe	2
6	4313·034	Ti	3	26	4371·442	Cr	2
7	4314·248	Sc	3	27	4373·415	Cr	1
8	4315·138	Ti	3	28	4373·727	Fe	2
9	4320·907	Sc	3	29	4374·981	Zr	0
10	4325·152	Sc	4	30	4376·107	Fe	6
11	4326·923	Fe	2	31	4379·396	V	4
12	4331·811	Ni	2	32	4379·927	Zr	0
13	4337·725	Cr	3	33	4383·720	Fe	15
14	4338·084	Ti	4	34	4385·144	Cr	2
15	4339·882	Cr	3	35	4388·571	Fe	3
16	4343·861	Fe	2	36	4390·149	V	2
17	4244·670	Cr	4	37	4395·201	Ti	3
18	4346·725	Fe	2	38	4396·008	Ti	1
19	4347·403	Fe	1	39	4400·555	Sc	3
20	4349·107	Fe	2	40	4400·738	V	1

The lines in the region common to all observers are :—

TABLE II.
Selected Lines in Common Region.

4220·509	Fe	3	4257·815	Mn	2
4225·619	Fe	3	4258·477	Fe	2
4232·887	Fe	2	4266·081	Mn	2
4241·285	Fe-Zr	2	4268·915	Fe	2
4246·996	Sc	5	4276·836	Zr	2

This gives a distribution of lines per element as follows :—

Element.	Number of Lines.	Element.	Number of Lines.
Fe	18	V	3
Ti	8	Zr	3
Cr	6	Mn	2
Sc	5	Mg	1
Ca	3	Ni	1

Some plates in these groups were taken in September 1910, but the series dealt with in the present paper begins with a set taken on December 22, 1910, while during the following year sets of equatorial plates were secured in every month (except April) up to and including October. Sixty of these plates were picked out for measurement, 12 of these covering the common region round λ 4250, while the 48 others represent 24 observations of the region λ 4300– λ 4400.

The measurements were carried out with a Zeiss comparator similar to that described by Professor Newall in *M.N.*, lxxv. p. 648. The stage holding the spectrogram is fixed on a slide which also holds a scale 100 millimeters long, divided to tenths of millimeters. The selected lines on the spectrogram are bisected in a fixed microscope, and the corresponding readings of the scale are read off to 0·0001 mm. in a second fixed microscope by means of a micrometer, on which one drum division corresponds to 0·001 mm. The slide is so made that it can be pushed by hand; when kept properly lubricated its smallest movement produced by end-on pressure of the hand is found to correspond so accurately to the smallest amount detectable by eye, that clamp and slow motion are unnecessary. Bisections by direct pressure of the hand on the slide are quite as accurate as those made with the help of the clamp and slow motion and take far less time.

For the purpose of these measurements trial has been made to eliminate the disturbing effects of photographic grain by inserting a cylindrical lens of the proper focal length between the eye of the

observer and the eyepiece of the microscope used for viewing the spectrograms. Both the bisecting wire and the spectral lines are viewed through the cylindrical lens which, whilst it leaves the sharpness of the bisecting wire unchanged, increases the appearance of homogeneity in the lines by obliterating almost entirely the visibility of the separate photographic grain at the edges. A note giving more details about this arrangement will be found in the same number of this Journal, p. 28.

It may be mentioned here that in a final comparison between the measurements with and without cylindrical lens the mean error (see below) is the same in both, while in the former case measurements can be more quickly made and, according to the writer's experience, with less fatigue to the eye.

Errors of scale have no influence on the displacements measured so long as the wire in the scale microscope is brought to coincidence with the same graduation mark on the scale for the two positions of a line. This safeguard is not troublesome, as the maximum amount of the shift is about 0.05 mm. and so equal to about half the distance between consecutive marks. It is only important to ascertain, as was done, that a complete turn of the drumhead corresponds accurately to 0.1 mm.

Each plate was measured twice, once red to right and once red to left. In each of these cases two independent settings were made on every selected line at each limb. A complete observation of the rotation shift on a selected line on one plate thus depends on eight settings. Such a single complete observation has been found to have a mean error of 2.8 drum divisions, corresponding to a velocity of 0.18 km. per second in the line of sight.

The reduction from displacement δR in drum divisions to velocity in the line of sight v in km. per second is done by multiplication by a factor containing the dispersion $\frac{d\lambda}{dR}$ as well as the velocity of light over the wave-length λ , as follows:—

$$v = \frac{V}{\lambda} \frac{d\lambda}{dR} \delta R,$$

the formula expressing Doppler's principle. The variation of the dispersion $\frac{d\lambda}{dR}$ from one end of the group to the other, owing to the spectrum not being quite normal, is not more than one in ten thousand. It was thus sufficient to take

$$\frac{d\lambda}{dR} = \frac{\delta\lambda}{\delta R} = \frac{\lambda_{40} - \lambda_1}{R_{40} - R_1}$$

for its value, where numerator and denominator denote the differences of the wave-lengths and scale-readings of the two lines at the extreme ends of the group. Moreover, the denominator

$R_{40} - R_1$ was found to have varied not more than 0.7 per thousand throughout the whole series of observations, and so the same value was used throughout for the factor $V \frac{d\lambda}{dR}$ in the reduction formula.

Corrections had to be made to the value of v thus found:—

1. The slit was always set a small distance h inside the limb (see fig. 1), where $h = 1.2$ mm. for a solar image of 172 mm. diameter. The velocities have therefore to be multiplied by a factor $\frac{R}{R-h}$, of which the value 1.014 varies not more than two in a thousand if the setting has been inaccurate to the amount of 0.2 mm. The seasonal variation in R , the radius of the image, amounts at most to 3 mm. Deducing the corresponding variation in the factor $\frac{R}{R-h}$, by taking account of the method of setting the image on the slit, we find this to be 0.001; it need not be taken account of.

2. The heliographic latitude of the centre of the Sun's disc D varying throughout the year, the equatorial velocities have to be multiplied with a different correcting factor according to the date. At the most this factor becomes 1.008.

3. The influence of light from other parts of the Sun besides that exactly on the slit has been neglected, as plates were only taken when atmospheric conditions were so that the intensity of the Sun's limb was 40 or 50 times as great as that of the "glare" in the immediate neighbourhood of the limb. It has been assumed that the integrated spectrum given by this "glare" cannot have been strong enough to cause any diminution in the value of the rotation shift recorded. This assumption seems justified by the fact that on a test plate taken at a date when the limb intensity was only three times that of the "glare" the measured shift was still quite two-thirds of its value on clear days.

4. The correction terms for the motion of the observer in the line of sight are not applicable here, as the measures are always differential. That, however, due to the observer's motion in the plane perpendicular to the line of sight, the term which changes the observed synodic values into sidereal ones, has to be applied. This has been done by using Dunér's table as given in his second publication (*Nova Acta Ups.*, ser. 4, I., No. 6) on pages 23 and 24.

In the following tables are grouped the results of measurements so reduced and corrected. The average has been taken for each line of that line's displacements on all the plates where it appears. Such an average is found to have a mean error of about 0.07 km. per second, which error would be a gauge of the accuracy of the observations throughout the year if we assume the equatorial velocity to be constant. Table III. gives these averages for the Cambridge region (24 observations each line), Table IV. for the region common to all observers (12 observations each line).

TABLE III.

Deduced Equatorial Velocity for each Selected Line in Cambridge Spectral Region.

	λ .	Element.	Km./sec.		λ .	Element.	Km./sec.
1	4299.149	Ca 3	1.847	21	4351.000	Ti 1	1.970
2	4300.732	Ti 2	1.812	22	4352.083	Mg 5	1.987
3	4302.085	Ti 2	1.965	23	4352.908	Fe 4	1.827
4	4302.692	Ca 4	1.877	24	4358.670	Fe 2	1.788
5	4307.907	Ca 3	1.802	25	4366.061	Fe 2	1.823
6	4313.034	Ti 3	1.839	26	4371.442	Cr 2	1.911
7	4314.248	Sc 3	1.802	27	4373.415	Cr 1	1.871
8	4315.138	Ti 3	1.931	28	4373.727	Fe 2	1.777
9	4320.907	Sc 3	1.856	29	4374.981	Zr 0	1.897
10	4325.152	Sc 4	1.801	30	4376.107	Fe 6	1.888
11	4326.923	Fe 2	1.859	31	4379.396	V 4	1.771
12	4331.811	Ni 2	1.847	32	4379.927	Zr 0	1.983
13	4337.725	Cr 3	1.859	33	4383.720	Fe 15	1.782
14	4338.084	Ti 4	1.868	34	4385.144	Cr 2	1.951
15	4339.882	Cr 3	1.829	35	4388.571	Fe 3	1.877
16	4343.861	Fe 2	1.888	36	4390.149	V 2	1.958
17	4344.670	Cr 4	1.879	57	4395.201	Ti 3	1.821
18	4346.725	Fe 2	1.862	38	4396.008	Ti 1	1.853
19	4347.403	Fe 1	1.944	39	4400.555	Sc 3	1.868
20	4349.107	Fe 2	1.953	40	4400.738	V 1	1.888

TABLE IV.

Deduced Equatorial Velocity for each Selected Line in Common Region.

λ .	Element.	Km./sec.
4220.509	Fe 3	1.787
4225.619	Fe 3	1.888
4232.887	Fe 2	1.956
4241.285	Fe-Zr 2	1.864
4246.996	Sc 5	1.796
4257.815	Mn 2	1.857
4258.477	Fe 2	1.904
4266.081	Mn 2	1.779
4268.951	Fe 2	1.769
4276.836	Zr 2	1.765

Grouping the values in Tables III. and IV. together, element by element, we get as result :

TABLE V.
Deduced Equatorial Velocity for each Element.

Element.	Number of Lines.	Km./sec.
Fe	18	1·857
Ti	8	1·883
Cr	6	1·883
Sc	5	1·847
Ca	3	1·845
V	3	1·870
Zr	3	1·915
Mn	2	1·819
Mg	1	1·989
Ni	1	1·848

where the second column gives the number of lines from which the resulting value is obtained. The mean error of these values varies from 0·018 to 0·024 km./sec. for the first four elements. We thus see that there is no strong evidence of systematic variation dependent on the assigned origin of the lines utilised in deducing these equatorial velocities; nor find we evidence of an abnormal velocity far outside the mean error in any one particular line (Tables III. and IV.). Similar negative results are got when grouping the lines together according to their intensities or their behaviour in arc and spark.

Accordingly it has been assumed at present that the deduced equatorial velocities exhibit no systematic variation depending on the assigned chemical origin of the various selected lines. It is then legitimate to take the mean of the velocities deduced from all the lines measured on each set of plates taken on a given date as fairly representing the value of the equatorial velocity on that date. The following tables exhibit the mean values thus obtained on 24 dates between December 22, 1910, and October 2, 1911 :—

TABLE VI.

Equatorial Velocity for each Date of Observation in Cambridge Region.

Date.	Plates.	Mean Displacement.	Km./sec.	ξ.
		mm.		
1910 Dec. 22	M 15, M 16	0.0521	1.772	12.61
1911 Jan. 14	M 23, M 24	547	1.858	13.22
28	M 28, M 29	504	1.726	12.28
30	M 32, M 33	605	2.043	14.54
Feb. 15	M 37, M 38	578	1.960	13.95
Mar. 22	M 42, M 43	554	1.883	13.40
May 6	M 48, M 49	577	1.943	13.83
9	M 53, M 54	566	1.909	13.59
10	M 56, M 57	496	1.689	12.02
11	M 59, M 60	555	1.872	13.32
12	M 62, M 63	534	1.806	12.85
17	M 71, M 72	532	1.799	12.80
22	M 74, M 75	552	1.859	13.23
30	M 80, M 82	546	1.841	13.10
July 11	M 83, M 84	548	1.851	13.17
13	M 89, M 90	557	1.879	13.37
24	M 92, M 93	535	1.812	12.90
29	M 98, M 99	558	1.887	13.43
Aug. 8	M 104, M 105	549	1.861	13.25
10	M 107, M 108	569	1.923	13.69
Sept. 8	M 110, M 111	566	1.918	13.65
23	M 112, M 113	552	1.875	13.35
25	M 116, M 117	551	1.870	13.31
Oct. 2	M 118, M 119	559	1.896	13.49

TABLE VII.

Equatorial Velocity for each Date of Observation in Common Region.

Date.	Plates.	Mean Displacement.	Km./sec.	ξ.
		mm.		
1911 Jan. 7	M 18	0.0497	1.723	12.26
28	M 30	535	1.844	13.12
Feb. 15	M 39	495	1.719	12.23
May 6	M 47	535	1.832	13.04
9	M 55	530	1.817	12.93
13	M 65	529	1.811	12.89
30	M 81	493	1.694	12.06
July 11	M 85	554	1.891	13.46
24	M 94	565	1.928	13.72
Aug. 10	M 106	521	1.792	12.75
Sept. 23	M 114	535	1.843	13.12
25	M 115	553	1.898	13.51

The third column in these tables gives the mean measured displacement in millimeters, the fourth the velocities in kilometers

per second, and the last the angle through which the Sun accordingly rotates in a day. It is obtained from the corresponding figure in the preceding column by multiplication with $\frac{N}{2\pi R} \times 360$, where N is the number of seconds in a day.

The mean error per plate varies from 2.0 drum divisions for the earlier winter observations to 0.9 for the best of the later ones, or from 0.09 to 0.04 km. per second in the velocities of the fourth column. We can take it to be about 0.05 km./sec. for most of the plates after March 22, 1911. We see that the range of velocities is such as this mean error leads us to expect.

The following table gives certain observational data in greater detail. It can serve to detect any systematic errors, if such have remained to an appreciable extent in the method of observation.

TABLE VIII.

Measured Displacements directly compared with Observational Data.

Date.	Plate.	Measured Displacement.	t_1 .	Plate.	Measured Displacement.	t_2 .	a .	da .	T.
1910. Dec. 22	M 15	mm. 0.0522	40	M 16	mm. 0.0521	65	13°7	0°0	8
1911. Jan. 14	M 24	559	115	M 23	538	33	8.3	0°0	8
28	M 29	479	70	M 28	528	34	12.6	0°0	14
30	M 32	561	60	M 33	645	86	5.9	0°0	12
Feb. 15	M 38	556	68	M 37	589	40	12.9	0°0	10
Mar. 22	M 43	573	60	M 42	536	40	6.7	+0.3	10
May 6	M 48	564	55	M 49	595	65	10.3	+0.2	1
9	M 53	556	55	M 54	577	60	10.5	+0.2	1
10	M 57	466	55	M 56	531	50	13.0	+0.5	1
11	M 60	535	88	M 59	575	83	13.1	+0.2	1
12	M 63	534	43	M 62	538	35	13.6	+0.5	1
17	M 72	546	51	M 71	529	40	14.1	+0.3	2
22	M 75	560	46	M 74	544	39	8.0	+0.1	2
30	M 82	553	920	M 80	538	900	15.2	+0.2	1
July 11	M 84	567	52	M 83	532	45	18.6	+0.5	2
13	M 90	553	27	M 89	564	20	16.9	+0.2	1
24	M 93	540	150	M 92	543	145	20.4	+0.4	1
29	M 99	549	60	M 98	594	42	22.0	+0.3	$\frac{1}{2}$
Aug. 8	M 105	551	105	M 104	547	100	18.6	+0.3	$\frac{1}{2}$
10	M 108	582	51	M 107	550	28	22.3	+0.1	$\frac{5}{8}$
Sept. 8	M 111	569	28	M 110	563	23	18.1	+0.4	$\frac{2}{3}$
23	M 113	552	70	M 112	551	43	11.1	+0.4	$1\frac{1}{3}$
25	M 116	550	39	M 117	552	48	12.2	+0.4	$1\frac{1}{2}$
Oct. 2	M 119	565	76	M 118	551	66	8.0	+0.4	1

In this table the second and fifth columns give the designations of the plates with, respectively, in the third and sixth columns the measured displacements, and in the fourth and seventh columns the intervals in minutes between the moment when the instrument was last touched (rotated for setting in the required position angle) and the beginning of the exposure. These two groups, of three columns each, concern respectively the plates taken at $\lambda 4325$ and those at $\lambda 4375$. The three last columns concern both the plates taken on the recorded date. They contain the temperature α inside the instrument at the time of the exposure, the change of temperature da in one hour, and the length T of each exposure in minutes. The table can be summarised in several different ways:—

TABLE IX.

Temperature inside Instrument compared with Results.

α .	Number of Plates.	Average measured Displacement.
		mm.
$5-10$	10	0.0563
$10-15$	22	543
$15-20$	10	554
$20-25$	6	557

TABLE X.

Rise of Temperature in one Hour compared with Results.

da .	Number of Plates.	Average measured Displacement.
		mm.
$0.0-0.1$	14	0.0552
$0.2-0.3$	18	557
$0.3-0.4$	16	544

TABLE XI.

Length of Interval after rotating the Instrument compared with Results.

t . (In Minutes.)	Number of Plates.	Average measured Displacement.
		mm.
$20-40$	10	0.0550
$40-80$	28	551
> 80	10	559

TABLE XII.

Length of Exposure compared with Results.

T . (In Minutes.)	Number of Plates.	Average measured Displacement.
		mm.
$8-14$	12	0.0551
2	6	544
< 2 and > 1	4	552
1	18	553
< 1	8	551

These tables show no sign of correlation between the observational data and the measured displacements. It may be concluded that none of these causes are active in producing systematic error in the results.

Latitude Plates.

Advantage was taken of the favourable weather conditions prevailing in Cambridge during May and June 1911 to get a series of observations all round the Sun in addition to the equator plates. On June 6 the heliographic latitude of the centre of the Sun's disc was zero, and on days near this date the reduction of the observed velocity in the line of sight to actual velocity at the limb, parallel to the plane of the equator, is much simplified, especially for points away from the equator. Several series of latitude plates were therefore taken from June 1 onwards during the fortnight of very clear weather succeeding. In this interval altogether 240 plates were taken, forming four complete groups or series of plates, each of which may be called a latitude series. I shall here deal with the measurements of one of these series, comprising sixty plates.

The observations were made in such a way as to be able to discriminate between the rotational velocities of the northern and southern hemispheres. In the method of observation in use at Cambridge, the two points on the limb to be compared together are alternately brought on the slit by moving the solar image itself across the slit plate. It is necessary, as we have seen, to obtain the exposures on these two points without rotating the instrument between the successive exposures. It is possible to do this not only for points situated at opposite ends of a diameter, but also, by the method of setting employed, already described on page 7 and shown in the diagram, for points situated at places 90° apart from each other on the Sun's limb. Accordingly the procedure which has been adopted in making these series of latitude observations has been as follows:—

The instrument is first set in such a position angle as would make the slit tangential to the Sun's limb at the desired latitude ϕ . A half exposure is then taken with the limb at this latitude illuminating the parts *dt* (see fig. 1) of the slit (slit tangential). The image is then moved across to a position with the limb at latitude $\phi + 90^\circ$ perpendicular to the slit (see again fig. 1) and a full exposure taken of the central part *s* (slit radial). Then the image is moved back to the first position and the completing half exposure taken at latitude ϕ . A new plate is now put in; the image is again moved to the position with the slit radial at $\phi + 90^\circ$. This time the first half exposure is given at this point, the image then moved to the point at $\phi + 180^\circ$ (slit tangential), a full exposure given; the image is then moved back to $\phi + 90^\circ$, slit radial, to complete the first exposure. A third plate is used to record the exposure at $\phi + 270^\circ$, slit radial, taken between two half exposures

at $\phi + 180^\circ$, slit tangential; and a fourth plate containing the spectra at $\phi + 270^\circ$ and again at ϕ serves to complete the round. Such a set of four plates having been obtained in the spectral region round $\lambda 4325$, the grating is then turned to the position with $\lambda 4375$ on the centre of the plate, and a similar set of four plates is obtained in that spectral region. In the case of ϕ equal to 0° , 30° , or 60° a third set of four plates is obtained in the spectral region round $\lambda 4250$, these latitudes having been assigned by the rotation committee of the Solar Union for observation in the spectral region common to all observers. When such a group of plates has been completed, the instrument is turned through 15° to a position with the slit tangential to the limb at latitude $\phi + 15^\circ$, and is left to settle down for half an hour to allow the effects of rotating the instrument, even through such a small angle, to disappear. A second group of plates round the Sun's limb is then obtained with this latitude $\phi + 15^\circ$ as starting-point, leaving the spectrograph untouched during the completion of this group, as in the case of the first. This procedure is repeated for latitudes at intervals of 15° up to $\phi + 75^\circ$. When this set is completed, exposures will have been made all round the Sun's limb at intervals of 15° . Altogether 60 plates will have been taken, 24 for each of the spectral regions $\lambda 4325$ and $\lambda 4375$, and 12 for the spectral region $\lambda 4250$.

It is the result of the measurement of such a latitude series of 60 plates which we will now discuss.

We must note at once the essential difference of the quantities observed in the usual method, adopted by other observers, with those observed here. In the case of observations at the ends of a diameter, as made by Dunér, Halm, and Adams, the displacement measured represents the difference of velocities

$$v_\phi - v_{(\phi + 180^\circ)},$$

which is equal to v_ϕ if symmetry is assumed not only round the Sun's axis, but also on either side of the equator. In the present case the displacement measured represents a difference of velocities

$$v_\phi - v_{(\phi + 90^\circ)}.$$

(The angles ϕ , $\phi + 90^\circ$, $\phi + 180^\circ$, etc., are in these formulæ, for convenience' sake, taken to be measured from the east end of the Sun's equator towards the North Pole and onwards all round the Sun.)

It is seen at once that in case the assumption of symmetry with respect to the equator is unjustified, the values of the latter quantity for ϕ equal to 15° etc., up to 75° , will be different from the corresponding values for ϕ equal to 195° etc., up to 265° , and a discrimination of northern from southern hemisphere can be affected.

The values of these differences of linear velocities are obtained from the mean measured displacements by multiplication with a factor in accordance with the formula representing Doppler's principle, exactly in the same way as in the case of the equatorial plates (see p. 11). The measurements have been made, also, with the same instrument and by the same method, while the means had been obtained from the measurements of the displacements of the same 40 lines (two plates) as given in Table I.

As to the corrections which have to be made, we find :—

1. The factor $\frac{R}{R-h}$ (see p. 12) has to be applied for correction

due to setting the slit a distance h inside the limb. We here assume that the displacement measured is the mean of the displacements of the two parts of the line exposed through the double opening $d\ell$ of the slit from the part exposed through the central opening s ; and we also assume that this mean accurately represents the displacement which would be found between the two central points which fall at the moment of the respective exposures on the centre of the slit. The first of these assumptions is entirely realised by the optical method employed in the measurement, of which the principle is to make every part of the line in the field contribute equally to the homogeneous image actually measured (see the note about the method of measurement). The second assumption is rigorously true in case of a sun rotating as a solid body. But a correcting term has to be applied to allow for the change of angular velocity with latitude. This correction, assuming the law of equatorial acceleration as found by earlier observers, is found on calculation not to exceed two parts in a thousand in our case of very short lines (1 mm. for the one spectrum, 3 mm. from top to bottom for the other, on a solar image of 172 mm. diameter). It can thus be neglected for the present, but must not be lost sight of in cases when a longer slit is used, or if the latitude law is found to be very different.

2. The heliographic latitude of the centre of the Sun's disk being for all the present observations less than $0^{\circ} 15'$ (see p. 18), no correction need be applied.

3. The influence of the "glare" is neglected on the same grounds as in the case of the equatorial plates.

4. The terms for reducing from synodic to sidereal values have to be evaluated directly by computation; we cannot use Dunér's tables, as the correction is different for v_{ϕ} and $v_{(\phi+90)}$. Here again the computation is much simplified by the Earth being in, or very near, the plane of the Sun's equator at the time of these observations.

The results thus obtained are incorporated in the following tables :—

TABLE XIII.

Latitude Plates, Cambridge Region λ 4300- λ 4400.

Date.	Plates.	$\phi, \phi+90.$	Mean Measured Displacement.	$v\phi - v(\phi+90).$ Km./sec.
			mm.	
1911 June 8	N 202, N 198	0°, 90°	-0'0274	-1'830
8	N 201, N 197	90, 180	- 254	-1'739
8	N 200, N 196	180, 270	+ 260	+1'742
8	N 199, N 195	270, 0	+ 261	+1'783
8	N 189, N 193	15, 105	- 320	-2'153
8	N 188, N 192	105, 195	- 244	-1'641
8	N 187, N 191	195, 285	+ 312	+2'102
8	N 190, N 194	285, 15	+ 170	+1'178
4	N 71, N 67	30, 120	- 331	-2'246
4	N 70, N 66	120, 210	- 128	-0'873
4	N 69, N 65	210, 300	+ 347	+2'346
4	N 72, N 68	300, 30	+ 153	+1'029
5	N 87, N 91	45, 135	- 330	-2'253
5	N 86, N 90	135, 225	- '0006	-0'062
5	N 85, N 89	225, 315	+ '0362	+2'453
5	N 88, N 92	315, 45	- '0003	+0'005
7	N 163, N 167	60, 150	- '0342	-2'327
7	N 162, N 166	150, 240	+ 133	+0'858
7	N 161, N 165	240, 330	+ 316	+2'165
7	N 164, N 168	330, 60	- 123	-0'795
7	N 159, N 155	75, 165	- 307	-2'095
7	N 158, N 154	165, 255	+ 213	+1'406
7	N 157, N 153	255, 345	+ 345	+2'333
7	N 160, N 156	345, 75	- 232	-1'525

TABLE XIV.

Latitude Plates, Common Region round λ 4250.

Date.	Plates.	$\phi, \phi+90.$	Mean Measured Displacement.	$v\phi - v(\phi+90)$ in km./sec.
			mm.	
1911 Jun. 8	N 206	0°, 90°	-0'0276	-1'863
8	N 205	90, 180	- 286	-1'960
8	N 204	180, 270	+ 256	+1'736
8	N 203	270, 0	+ 315	+2'143
4	N 75	30, 120	- 331	-2'271
4	N 74	120, 210	- 113	-0'787
4	N 73	210, 300	+ 336	+2'302
4	N 76	300, 30	+ 117	+0'812
7	N 171	60, 150	- 321	-2'219
7	N 170	150, 240	+ 173	+1'121
7	N 169	240, 330	+ 344	+2'365
7	N 172	330, 60	- 138	-0'899

The third columns contain the latitudes, putting ϕ first and then $\phi + 90$, at which the middle exposure was taken, and measuring ϕ from the eastern end of the equator towards the north, all round the Sun. With regard to the sign in the fourth and fifth columns, a velocity in the line of sight has been taken as positive when the source is travelling away from the observer, in accordance with the usual practice. The signs in the two columns are the same except in the case of $\phi = 315^\circ$ (Table XIII.), where the correction term necessary to transform into sidereal velocities, which is always an *additive* term, was of an order to change the small negative value into a small positive one.

The mean errors of the values in the last column in Table XIII. vary from 0.05 to 0.09 km. per second.

We will now show what information about the latitude law the material so obtained can afford. It is evident that, without making certain assumptions, it is not possible to deduce the values of v_ϕ itself in any direct way from these differences $v_\phi - v(\phi + 90)$. Indeed, without any assumptions there would be an infinite number of solutions of

$$v_\phi = f(\phi)$$

when only the function

$$F(\phi) = f(\phi) - f(\phi + 90)$$

is given.

One assumption, however, we may safely make at once, viz. that of symmetry round the solar axis; disregarding local disturbances, the tangential velocity should be the same all round a solar parallel. Making no other assumptions at all about the form of $f(\phi)$, we can elicit some important information in the following way:—

Remembering that we are dealing with differences of velocities at points 90° apart, we add these differences together per quadrant. We then shall have got the difference of the sum of the velocities in the first (N.E.) quadrant from the sum of the velocities in the second (N.W.) quadrant; similarly the difference between the sum of the velocities in the second (N.W.) quadrant and the sum in the third (S.W.) quadrant; the sum of the velocities in the third (S.W.) minus that sum in the fourth (S.E.), and finally the sum in the fourth (S.E.) minus again the sum in the first (N.E.) quadrant. Now it is evident that, in case of asymmetry between the northern and southern hemispheres, the difference of the sums in N.W. and S.W. quadrants will have a definite value; while our assumption of symmetry round the axis entails that we should find the same value, different from zero, for the difference between the sums in the N.E. and S.E. quadrants. Then again if we take account of the signs of the velocities, the difference between the sums in the N.E. and N.W. quadrants really becomes a sum of velocities over the whole northern hemisphere; just as the difference between the sums of S.W. and S.E. becomes a similar summation over the whole southern hemisphere. We have thus a third independent comparison of the two hemispheres.

Analytically this way of dealing with the material can be put as follows:—

Adding together the observed values of

$$F(\phi) = f(\phi) - f(\phi + 90)$$

for $\phi = 15^\circ, 30^\circ$, etc., up to 75° , we have,

$$\sum_{\phi=15}^{\phi=75} F(\phi) = \sum_{\phi=15}^{\phi=75} f(\phi) - \sum_{\phi=15}^{\phi=75} f(\phi + 90),$$

or

$$\sum_{\phi=15}^{\phi=75} F(\phi) = \sum_{\phi=15}^{\phi=75} f(\phi) - \sum_{\phi=105}^{\phi=165} f(\phi) \quad . \quad . \quad . \quad (1)$$

The assumption of symmetry with respect to the axis can be expressed thus, having regard to the sign:

$$f(\phi) = -f(180 - \phi) \quad . \quad . \quad . \quad (2)$$

Hence equation (1) becomes

$$\sum_{\phi=15}^{\phi=75} F(\phi) = 2 \sum_{\phi=15}^{\phi=75} f(\phi) \quad . \quad . \quad . \quad (3)$$

Similarly we find for the summation of $F(\phi)$ for $\phi = 195^\circ, 210^\circ$, etc., up to 255° ,

$$\sum_{\phi=195}^{\phi=255} F(\phi) = -2 \sum_{\phi=195}^{\phi=255} f(\phi) \quad . \quad . \quad . \quad (4)$$

having here transformed the right-hand side by means of equation (2) again, so that it contains $f(\phi)$ on the eastern half of the Sun only.

Adding now $F(\phi)$ for $\phi = 105^\circ$, etc., to 165° , we find

$$\sum_{\phi=105}^{\phi=165} F(\phi) = \sum_{\phi=105}^{\phi=165} f(\phi) - \sum_{\phi=105}^{\phi=165} f(\phi + 90) = \sum_{\phi=105}^{\phi=165} f(\phi) - \sum_{\phi=195}^{\phi=265} f(\phi),$$

or, with the help of equation (2), again to deal with $f(\phi)$ only on the eastern half of the Sun,

$$\sum_{\phi=105}^{\phi=165} F(\phi) = \sum_{\phi=105}^{\phi=165} f(\phi) - \sum_{\phi=15}^{\phi=75} f(\phi) \quad . \quad . \quad . \quad (5)$$

while finally we also have

$$\sum_{\phi=285}^{\phi=345} F(\phi) = \sum_{\phi=285}^{\phi=345} f(\phi) - \sum_{\phi=285}^{\phi=345} f(\phi + 90),$$

or

$$\sum_{\phi=285}^{\phi=345} F(\phi) = \sum_{\phi=285}^{\phi=345} f(\phi) - \sum_{\phi=15}^{\phi=75} f(\phi) \quad . \quad . \quad . \quad (6)$$

The left-hand sides in these four equations (3), (4), (5), and (6) are independently known from the observations. Equations (5) and (6) therefore give two independent determinations of the value of

$$\sum_{\phi=15}^{\phi=75} f(\phi) - \sum_{\phi=285}^{\phi=345} f(\phi),$$

while by adding (2) and (3) we get another independent determination of this value. It is easily seen that in case of symmetry of the arrangement of the velocities with the latitudes with respect to the Sun's equator, the value of this form should be zero, so that any systematic departure from the value zero may be taken as proof of dissimilarity in the two hemispheres.

If now, to execute this numerically, we put

$$E = \sum_{\phi=15}^{\phi=75} f(\phi) - \sum_{\phi=285}^{\phi=345} f(\phi)$$

and substitute in the left-hand sides of the equations which determine E the observed values of $F(\phi)$ from Table XIII. (special Cambridge region), we get for equations (5) and (6) :—

$$E = - \sum_{\phi=105}^{\phi=165} F(\phi) = - 0.312 \text{ km./sec.}$$

$$E = - \sum_{\phi=285}^{\phi=345} F(\phi) = - 0.108 \text{ km./sec.}$$

while from (2) and (3) we get

$$2E = \sum_{\phi=15}^{\phi=75} F(\phi) + \sum_{\phi=195}^{\phi=265} F(\phi) = - 0.225 \text{ km./sec.}$$

It is seen that in these three independent determinations E is not only consistently negative, but also of much the same order of magnitude. Its mean value would be

$$E = - 0.186 \text{ km./sec.,}$$

giving double weight to the third value which depends on double the number of observations.

The observations thus afford consistent evidence of asymmetry between northern and southern hemisphere, E being negative meaning that the sum of the velocities at certain latitudes in the southern hemisphere is larger than the sum of the velocities at those latitudes in the northern hemisphere.

We can now treat the observations by the method of least squares. Assuming the ordinary latitude law to hold, as it has been determined by most observers, but distinguishing in accord-

ance with what we have just found between northern and southern hemisphere by variation of the second constant, we have

$$v_{0 < \phi < 180} = (a - b \sin^2 \phi) \cos \phi$$

and

$$v_{0 > \phi > 180} = (a - (b + b\epsilon) \sin^2 \phi) \cos \phi.$$

With these we then form $v_\phi - v_{(\phi+90)}$ for all the latitudes round the Sun, and equate the forms thus found to the values from Table XIII. Including this time the observations for $\phi = 0^\circ, 90^\circ, 180^\circ$ and 270° , which had not been taken account of in the preliminary investigation as they cannot afford any evidence about symmetry or asymmetry with respect to the equator, we get twenty-four equations in which a , b , and $b\epsilon$ are the three unknowns. It is found that these are best dealt with by adding them as in the preliminary investigation per quadrant, but now six by six, starting at $\phi = 0^\circ$ up to $\phi = 75^\circ$. We then get the following four equations of condition:—

$$\begin{array}{rcl} +7.596a - 2.502b & -12.904 & = 0 \\ a & -1.251b\epsilon - 2.051 & = 0 \\ -7.596a + 2.502b + 2.502b\epsilon + 13.141 & = 0 \\ -a & -1.251b\epsilon + 1.675 & = 0 \end{array}$$

in which the coefficients of the unknowns are entirely found from the numerical values of trigonometrical functions of the different angles of latitude, while the known terms depend only on the observations. Solving these by method of least squares, we get for the solutions and their weights:—

	Weight.
$a = +1.863$ km./sec.	2.00
$b = +0.513$ km./sec.	0.21
$b\epsilon = -0.123$ km./sec.	6.26

This solution confirms, as was to be expected, the previous result. Although the second constant b comes out with a small weight and so is very uncertain, the difference in this constant between the two hemispheres comes out with a big weight and with a sign such as to indicate that the velocities in the southern hemisphere are larger than those in the northern. It should also be noted that the value of the constant a , which, as is seen from the formulæ at the top of the page, represents the equatorial velocity, agrees excellently with the equatorial velocities found directly (Table VI., page 15).

Fig. 2 gives a graphical representation of these results. The axis of abscissæ contains the latitudes all round the Sun, while the ordinates are km./sec. The dots represent the observed values of $v_\phi - v_{(\phi+90)}$; their size in a measure represents the accuracy of their determination. The curve represents the least squares solution; the asymmetry between the two hemispheres appears in the

evident difference between the negative and positive parts of the curve.

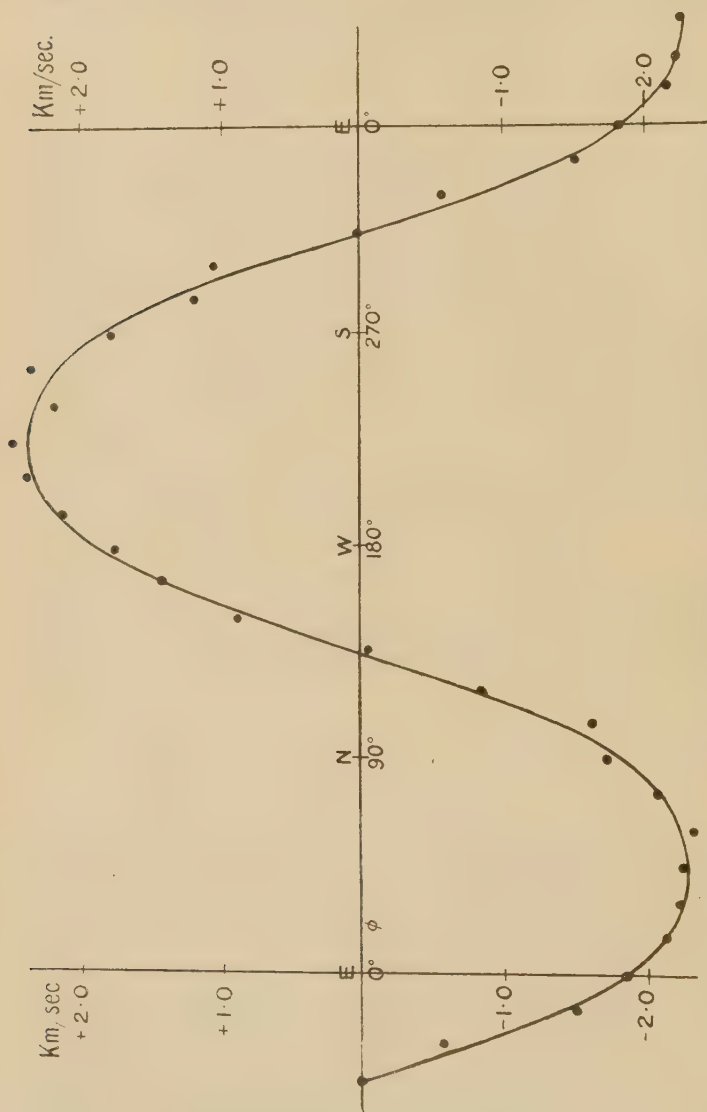


FIG. 2. $v_\phi - v_{\phi+90}$ for ϕ from 0° to 360° . The dots represent the observations, the curve their least squares solution given in the text.

CONCLUSIONS.

1. The equatorial velocity is found to be 1.86 km. per second on the average throughout the year 1911. This value is consider-

ably lower than that of about 2.01 km. found by other observers. There are no indications, however, of any systematic error in the method of observation. In particular there is no evidence that the fact that the exposures on the two points of the limb compared are not taken simultaneously has any influence on the amount of the displacement recorded. It will be of interest to see whether observations made elsewhere during 1911 agree in giving a reduced equatorial velocity. If they do not it might yet seem desirable to take another series of observations with the same instrument, but then equipped with an arrangement for simultaneous exposures.

2. The measurements on the series of equatorial plates reveal no instance of peculiar behaviour in the case of any single line out of the forty chosen in the spectral region studied, when we consider their mean errors. Neither is there any one element of which the lines show a displacement consistently larger or smaller than the normal. It should be mentioned that all the elements of which more than one line was measured have an atomic weight not far from 40 or 50.

3. The method of comparing the velocities at points on the limb situated 90° apart—a method which was easily applicable in the case of the present instrument—has proved of value in the detection of a difference between the rotation law in the northern and southern hemispheres of the Sun. There seems to be no doubt that this difference exists in the case of the single series of observations discussed in the latter part of the present paper. However, three more such series of plates exist taken under exactly similar circumstances, by the same method, and practically at the same time. It seems important that measurements of these further series should be made to see whether they confirm the result indicated by the first. These further observations will then, together with the series here presented, afford enough material for a more searching inquiry into the nature of the latitude law that held good at the date of observation (June 1 to 15, 1911).

I would like to express my sincere gratitude to Professor Newall for all he has done in connection with the present work, from the moment when he suggested my taking it up to his now communicating these results to the Society. Without his constant help and his kindness in putting all the resources of the Astrophysical department of the Cambridge Observatory, including its newest instruments, at my disposal, this research could never have been undertaken.

I also wish to thank Mr. L. J. Stanley, who assists generally in the solar work at the observatory, for his help with the observations as well as in the preliminary work with the instrument.

Note on a Method of measuring Spectrograms with the help of a Cylindrical Lens. By J. B. Hubrecht, M.A.

In the measurement of the positions of lines on high-dispersion photographs of the solar spectrum by the method of setting a fine micrometer wire to coincidence with the centre of the lines the appearance of the grain on the plate often causes difficulty. To those observers whose natural inclination lies more in the direction of slow measurement than in that of the quick and almost instantaneous setting on what appears at the first glance to be the centre of the lines the difficulty is much emphasised. Indeed it may almost be said that in the high-dispersion plates measured for the determination of the Sun's rotation, dealt with by the writer in the previous paper, the quick method is the only one that would give satisfactory results.

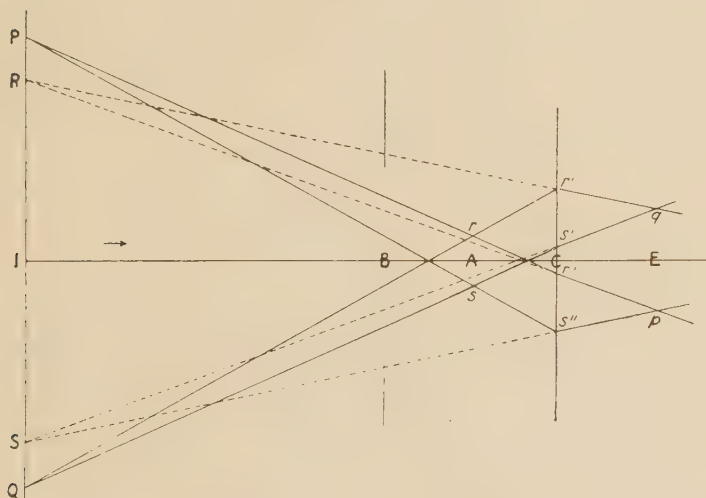
It seemed important, therefore, to devise a method of measurement by which these difficulties caused by the photographic grain would be minimised. The following solution of the problem was, after some experimenting, adopted by the writer in the solar rotation work.

A very short focus cylindrical lens is placed between the eye-lens of the microscope and the observer's eye, with its axis parallel to the dispersion on the plate. The position of this lens is such that, were it an ordinary lens of the same focal distance, an image of the exit-pupil of the microscope, *i.e.* of the *image* of its objective existing just outside the eye-lens, would be formed at the distance of normal vision from the observer's eye. A field of equal illumination with a diameter proportional to the diameter of the objective would in that case be seen by the eye, whatever the object on the stage. In the case of the cylindrical lens the rays in the plane perpendicular to its axis, in which plane the lines of the spectrum lie, proceed exactly in this way; while in the plane of the axis the rays proceed as if there existed no cylindrical lens. The result is that we see a longitudinal strip of light in the field, of a width dependent on the diameter of the objective, perfectly homogeneous in the direction perpendicular to the axis of the cylindrical lens, but showing the variations of light and darkness corresponding with their distribution on the photographic plate in the direction parallel to this axis, the direction of the dispersion. In other words, we see in the field the spectrum as it is on the photographic plate, integrated over its original width, and having now a width dependent only on the dimensions of the microscope. Every point of a line originally in the field of the microscope contributes equally to every point of the new homogeneous line, sharply ended at top and bottom as it now appears in the centre of the new field.

To make this point quite clear the following diagram may serve to show the path of the rays in the plane perpendicular to the axis of the cylindrical lens and to the dispersion on the plate.

PQ is the (virtual) image of the line as formed by the micro-

scope. After having passed out of the ordinary eyepiece at B, the pencils from the points $P \dots Q$, all having a definite width dependent on the microscope dimensions only, pass through rs , the exit-pupil of the instrument. The cylindrical lens is placed at C at such a distance that all rays passing through the point r after having gone through the lens proceed in a pencil $r'q$, $r''p$ as if they came from a point R at normal vision distance, all rays from s as if they came from S. The eye placed at E perceives a line RS. The light from a point R of such a line in reality is a superposition of light coming from every point $P \dots Q$ of the original image, similarly the light from any other point of the line RS, which accordingly presents a perfectly homogeneous appearance.



In practice the dimensions and focal distance of the cylindrical lens are determined by the original geometrical dimensions of the microscope and by the conditions that while IE must be equal to the distance of normal vision, the width of the new exit-pupil pq must not be greater than the diameter of the pupil of the observer's eye.

Calculations showed that a cylindrical lens of focal length 12.5 mm. would best suit the microscope used by the writer for his measurements. It was fixed to the eyepiece by means of a sliding collar which allows for focussing, while the adjustment of the axis of the cylindrical lens is provided for by the possibility of rotating the whole eyepiece in the tube of the microscope.

Each plate is set in the right position on the stage by observing through an ordinary eyepiece the parallelism of a few lines to the micrometer wire. The eyepiece containing the cylindrical lens is then put in and the measurements can proceed. As has been

shown, whatever the original distribution of the light in a line in the direction of its length it will appear centrally in the new field. If, for example, a plate is under observation containing several non-contiguous strips of spectra, the appearance through the new eyepiece will all the same be that of a single spectrum of definite width in the centre of the field. It is evident that in such a case—indeed in any case in which the observed spectrum does not occupy the whole of the original field—it is necessary to provide a diaphragm. Without such a diaphragm the white light wherever it appeared in the original field would be superposed on the image of the spectrum observed through the cylindrical lens and much diminish its visibility.

In the measurement of the solar rotation plates, dealt with by the writer in the previous paper, the use of such a diaphragm results in an added advantage. In these plates the displacement to be measured for each individual line is the displacement between such a line on the central strip of spectrum originating from one point on the Sun's limb, and the same line as it appears on the two contiguous strips which originate from the other point on the Sun's limb, of which the velocity in the line of sight is to be compared with that of the first. A movable diaphragm is now so placed immediately above the plate, but without touching it, that either only light from the one single spectrum, or only light from the double spectrum, can proceed through the microscope. In both cases a single central spectrum is viewed in the eyepiece; on moving the diaphragm from the one position to the other while keeping the eye to the instrument, all that is seen to change is the position of the lines. They are shifted to the amount of the displacement to be measured. It is thus possible to make the settings on the two positions of the line consecutively, without being biassed in setting on one position by the appearance of the line in the other. Indeed the bias remaining is all to the advantage of accurate measurement. Supposing a certain line to have in general a real unsymmetrical structure, being, for example, decidedly sharper on the violet side than on the red, the observer will tend to set the micrometer wire in a certain way with respect to this structure. The diaphragm being moved and the line shifted, the tendency naturally is to set the wire in the same way with respect to this structure of the line as it now appears. On the other hand, supposing a spurious structure caused by individual strong grains contributing unduly to the integrated image still existing, the constant moving from one image to the other and *vice versa* will at once enable the observer to distinguish the true appearance of a line from the accidental appearance caused by remaining photographic imperfections.

The mean error of the measurement of a displacement has been determined both without and with the cylindrical lens, by measuring a certain plate several times. In the former case it amounted to 2.8 drum divisions, or 0.0028 mm.; in the latter to 0.0027 mm.

It is not claimed that the method is generally more accurate than the usual one. But its advantages may be summed up as the following:—

1. The lines are in general more homogeneous and so easier to bisect. The bisections, as a result, are made with less hesitation; the measurement of a plate is completed more quickly.

2. The fact of having to deal with these homogeneous lines has, at any rate in the case of the writer, the physiological effect of making the measurements less fatiguing to the eye. It was possible, in his case, not only to measure more material per hour, but also to measure more hours per day, so that his total output of measurements per day increased more than threefold.

In the case of the rotation work there is, moreover, the advantage, already mentioned, and caused by the necessary use of a diaphragm, of consecutive settings on the two positions of the same line.

*Note on the Oxygen Triplet in the Infra-Red
of the Solar Spectrum. (Plate I.)*

(Communicated by the Astronomer Royal for Scotland.)

In 1883 Professor Piazz Smyth in some researches in the infra-red spectra of various elements noticed a line in the spectrum produced in the oxygen vacuum tube about $\lambda 7775$.* In 1896 Runge and Paschen found this line to be the strongest member of a triplet belonging to the principal series of the oxygen spectrum and pointed to its apparent identity with three lines in the solar spectrum shown in Higgs' map.† The lines were difficult to see and the agreement of the measures was not absolute, but since this region of the spectrum is very bare of lines, and since the circumstances of the discharge in the vacuum tube differ widely from what may be supposed to be the circumstances of the atmosphere, the authors concluded that the identification was reliable and that the oxygen which produced the lines was in the atmosphere of the Sun. As a check upon this conclusion they referred to Mr. F. McClean's photographs of the region with high and low Sun, but left the final decision to a future examination by others of the presence or absence of a relative Doppler displacement of these lines when taken from the east and west limbs of the Sun.

At that time there was no clear evidence of the presence of oxygen in the Sun. Since then the recognition of the flutings of titanium oxide in sun-spots has put the question in a different position, but there remained the point whether oxygen was found in a free state in the absorbing layer.

In 1911, on the completion of the investigation of the solar

* Cf. *Trans. Roy. Soc. Edin.*, xxxii. pp. 238, 446.

† *Astrophys. Jour.*, iv. p. 317.

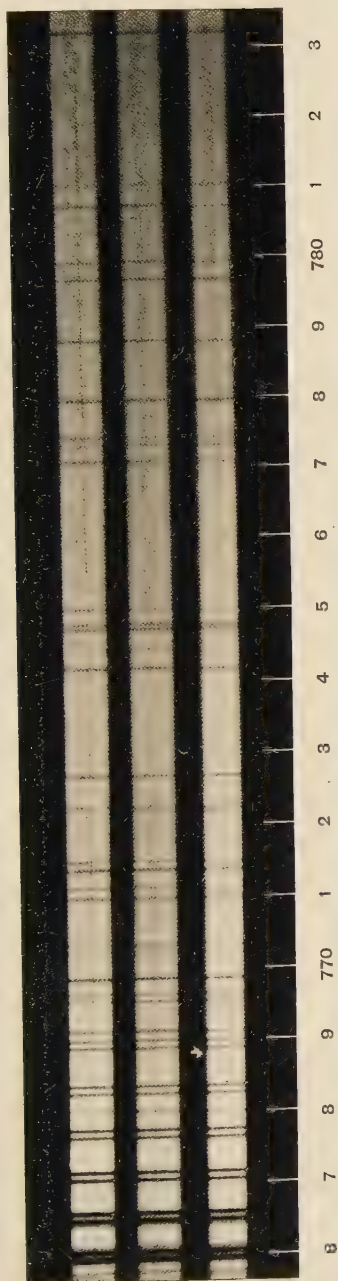
rotation then in progress at Edinburgh Observatory, we fitted a new arrangement of reflecting prisms to the slit-end of the spectro-scope to permit simultaneous examinations of both limbs of the Sun. By this arrangement light upon the middle of the slit comes from one limb and that upon the top and bottom of the slit from the opposite limb, and different latitudes upon the disc can be taken by revolving the set of prisms about a horizontal axis. To make a thorough test of this appliance we turned upon the oxygen triplet.

It was necessary to proceed by photography, because we never succeeded in seeing any lines beyond about $\lambda 7700$, still less in measuring their displacement. At first spectrum panchromatic plates were used. The sensitiveness of these plates is low in the region in question, and the results were poor. Still, on September 10, 1911, with an exposure of $2\frac{1}{2}$ hours, a plate was obtained which showed beyond any reasonable doubt that the lines were displaced. The images were too weak to allow of successful reproduction, but I showed an enlarged copy of it to several members of the Society, who confirmed my judgment as to the reality of the displacement. But $2\frac{1}{2}$ hours was already too long an exposure. The state of the sky changes rapidly at Edinburgh, and of course any haziness which may come over the sky confuses the effect that is being examined. It is doubtful too whether throughout the heat of the day the relative positions of the slit and grating, separated by 16 feet, in a room just below the roof, may not change enough to spoil the result. To get more sensitive plates a portion of alizarine blue was obtained and the plates dyed. We followed Blythwood and Scobell's directions,* employing Wellington orthochromatic plates, and found them to work as reliably as the authors claim and without any difficulty. The plates keep fairly well for one or even two days after dyeing, but are best used at once.

While the investigation was in progress the Annual Report of Mount Wilson Observatory for 1911 was received, where it is announced (p. 169) that Mr. St. John had taken up the question with the 30-foot spectrograph and concluded that the lines in question are of purely solar origin, the displacements shown being closely those given by known solar lines. The date is not given, but it was presumably early in the year 1911, because the fact is mentioned in Abbot's *The Sun*, which is dated 1911 July 10.

The early part of this year was occupied with some readjustments of the spectroscope. The summer was particularly unfavourable. For three months together it was not possible to obtain a single plate. At length in September some good days were found and a very fair plate was obtained on September 23, and one still better on September 27. The reproductions and measures below are from the latter. The spectrum was in the first order and the exposure was one hour.

* *Astrophys. Jour.*, xxiv, p. 125.



REGION OF SOLAR SPECTRUM SHOWING DOPPLER DISPLACEMENT OF OXYGEN TRIPLET AT 7772-7775.
ROYAL OBSERVATORY, EDINBURGH, 1912 SEPTEMBER 27.

Outer spectra from West limb; middle spectrum from East limb.
First Order. Exposure 1^h; enlarged three times; Wellington Orthochromatic Plate, dyed with Alizarine Blue.

Mr. St. John speaks of the identification with oxygen as being not quite conclusive, because the measures of the lines produced in the laboratory are not exact enough. All our negatives give the persistent impression that the least refrangible line does not behave in quite the same way as the others. By visual examination I should say it appears, relatively to the others, not to be displaced. It is weak and hazy and difficult to measure, but the measures seem to negative this impression and mark it as well as its two companions as definitely solar.

The measures were made by Mr. J. Storey. As the field of the micrometer at present in use does not show the whole of the three spectra, and as it was not permissible to shift the plate during measurement, it was first set to show the top and middle spectra and the measuring web taken first across the top and middle images of each spectrum and then backwards across the same; the settings upon each image of each line were wholly independent of one another; then the plate was shifted to show the middle and bottom spectra and the same step repeated. The mean of the two differences (*middle - top*) and (*middle - bottom*) gives the measure. Along with the triplet in question three other lines were measured, as a check, viz. λ 7696.1, 7697.2, 7699.2, as read from Higgs' map. Of these the first two are manifestly a pair of the A series and may therefore be expected to show zero displacement, while the third, by examination of the photograph, is equally clearly solar. Three measures were made as follows, the unit being $0^{\text{mm}}.001$:—

Line.	Middle-Top.			Middle-Bottom.			Mean.	
	(1)	(2)	(3)	(1)	(2)	(3)		
7696.1	-18	-1	0	+1	+12	+8	0	
7697.2	+10	-23	-1	+14	-9	+1	-1	
7699.2	-29	-18	-27	-33	-30	-30	-28	
Triplet.	7772.3	-63	-40	-55	-39	-29	-36	-44
	7774.4	-43	-28	-40	-44	-43	-46	-41
	7775.7	-25	-17	-26	-30	-24	-24	-25

The corresponding computed value of the displacement is 33, which is reasonably near the mean of the measures of the four solar lines above.

The Transit of Mercury in the Year 1707.

By G. van Biesbroeck.

(Communicated by the Secretaries.)

The transit of Mercury over the Sun on the 5th May 1707 (24th April, old style) happened under most unfavourable circumstances of visibility for central Europe. The conjunction took place near midnight 5th–6th May. Most of the astronomers placed so little confidence in the predictions of that epoch that they hoped yet to see at least a part of the passage.

Kirch* at Berlin looked for it already two days before the real conjunction, and continued his search an entire day after this moment. From most parts of Europe, France, Italy, Germany the reports were all negative, and during many years it was believed that nobody had had a view of the rare phenomenon.†

Only in the year 1723 Delisle communicated to Halley a letter where he refers to an unpublished observation of the transit by Roemer at Copenhagen ‡ in the terms: “Hodie sexto Maii hora matutina 4^h 19^m spectabatur Mercurius in extremo margine Solis jamjam exiturus; altus supra imum solis marginem $\frac{1}{4}$ diametri solaris et ad sinistrum in Tubo (sc. invertente). Accuratus haec determinare non licuit ob moram nimis brevem.”

Baily, while examining in 1835 Flamsteed’s manuscripts at the Royal Observatory at Greenwich, found some letters by Abraham Sharp,§ who wrote that he had a view of the end of the transit. Observing at Little Horton, near Bradford, he noted the exit of the planet at 16^h 52 $\frac{1}{2}$ ^m true local time. To the account of this uncertain observation, made under unfavourable circumstances, Flamsteed’s former assistant subjoined a diagram of the appearance of the phenomenon.

Comparing Sharp’s observation with Roemer’s, Baily discussed their contradictory results, especially as to the position of the vanishing planet upon the Sun’s disc and its latitude at this moment. He explains the inconsistency of the two results by the assumption that there had been a mistake in the interpretation of Roemer’s words, and that the expression “in Tubo invertente” did not apply to “supra imum solis marginem” but only to “ad sinistrum in Tubo.”

The publication of Roemer’s recently discovered *Adversaria*|| having brought to light—among other interesting facts concerning

* G. Kirchii, “De Mercurio in Sole, anno 1707,” *Miscellanea Berolinensia*, 1710, p. 218.

† J. Papken, *Incursus Mercurii in discum Solis*, p. 4. Gryphiswaldiae, 1720.

‡ E. Halley, “An Account of the Appearance of Mercury passing over the Sun’s Disk,” *Phil. Trans.*, No. 386, vol. xxxiii. pp. 228–238. London, 1726.

§ Baily, “Some Account of the Transit of Mercury over the Sun, observed by Mr. Abraham Sharp, in the year 1707,” *Monthly Notices of the R.A.S.*, vol. iii. pp. 105–107 (1835).

|| Th. Eibe and K. Meyer, *Ole Rømers Adversaria*. Kjøbenhavn, 1910.

the great Danish astronomer, *e.g.* that he was the real inventor of the Horrebow-Talcott method of determining the latitude and that he already employed Mayer's formulas for the correction of his newly invented transit instrument—an original statement as to the questionable observation, it seemed worth while to me to review the subject more attentively.

Roemer made extensive calculations for predicting the circumstances of the transit, notably the time of the conjunction and the position of the apparent orbit of the planet. But I refer here more especially to the paragraph on p. 227 of the new edition: "Circa Mercurium in Sole die 6 maji 4^h 19^m mane observatum Pilenburgi . . ." Roemer illustrates his computations and observations by a diagram reproduced in facsimile by the editors of the *Adversaria*. It bears the inscription: "Observatio Mercurii exeuntis ex Sole 1707 die 6 Maji mane H 4^h 20' inter undulas crispatis Solis."

The indicated position angle of the planet, which I have roughly measured on the diagram, is 306°, nearly accordant to the angle (300°) deduced from the words first reported by Halley; according to Baily the angle would have been 240°. This suffices to prove that Baily's assumption of the interpretation of the original text does not hold good. But there is more.

From Newcomb's Tables of Mercury and the Sun I have deduced the following elements for the geocentric transit:—

5th May 1707.	{	Conjunction in longitude	. 11 ^h 28 ^m ·3	Greenwich M.T.
		Heliocentric latitude	. + 1'·3	
		3rd contact	. 15 ^h 26 ^m ·4	„
		4th „	. 15 29 ·4	„
		Equation of true time	. + 3·7	

Roemer's observation was made at Pilenburg, the lonely place some ten miles west of Copenhagen where he had built his Tusculanean Observatory and set up his Rota Meridiana and Prime Vertical Transit. In his discussion of Roemer's Triduum, Galle* adopted for the position of this place:—

Longitude, 0^h 39^m 57^s·5 east of Paris.

Latitude, 55° 40' 53".

With these data I find the following circumstances for the exit at Pilenburg:—

3rd contact—

15^h 26^m·8 Greenwich M.T. = 16^h 19^m·8 true time Pilenburg.

4th contact—

29^m·8 Greenwich M.T. = 22^m·8 true time Pilenburg.

Angle from the zenith 277°.

According to the *Adversaria* the planet had left the Sun at 16^h 19^m or 20^m. This is a fair coincidence with the modern

* J. G. Galle, *Olai Roemerii triduum . . . reductum et cum tabulis comparatum*, Berolini, 1845.

computations; we ought to remember that, the Sun having risen only ten minutes before this moment, the seeing was very bad and consequently the planet was lost from view somewhat too early.

Roemer was little satisfied with this observation; he writes (p. 230): "Expectandum quid alibi sit determinatum ubi forte melius observatum. nam nobis certe subito punctum egressus in Sole ob vapores horizontis admodum crispato margine obiter saltem determinare licuit." Therefore, the fact that the indicated position of the planet's exit was nearly 25° in error is not very surprising. The words quoted explain perhaps why Roemer neglected to publish his results.

The computations made by Halley had placed the planet too far to the south; the error of Roemer's observation put it somewhat too northward; thus the discordance between the two appeared unusually large, so that Halley insinuated that the observation at Copenhagen was not made by Roemer himself but by "some person less acquainted with matters" (*Phil. Trans.*, *ibid.*, p. 238). The publication of the *Adversaria* makes it quite sure that Roemer himself was the observer.

As for what was seen by A. Sharp it is strange that Baily paid no attention to the time of that presumed observation: Little Horton being $7^m.1$ west of Greenwich and Pilenburg $0^h 49^m.3$ east, Sharp's time $16^h 52^m.1_2$ corresponds to $17^h 42^m$ true time Pilenburg. The planet was nearly $1\frac{1}{2}$ hours off the Sun when the Horton observer thought that he saw it. An error in the inscription of the time cannot better explain this inconsistency, for I find that at the last contact, computed above, the Sun was still more than one hour below the horizon of Little Horton. The end of the transit could not be seen at that place.

This result appears the more probable when we remark how doubtful Sharp's indications were and how prudently he reported what he had seen. It is curious to remark incidentally that at Little Horton the entrance of the planet had been visible for some minutes before sunset (5th May) but the sky was cloudy.

In conclusion, I find that Sharp's observation was spurious and that Roemer was the only person who saw the transit of Mercury in 1707, although his rough observation was but of poor value.

Uccle:

1912 October 31.

*Corrections to the New General Catalogue resulting from the
Revision of Sir William Herschel's three Catalogues of Nebulae.*
By J. L. E. Dreyer, Ph.D.

The following list has been put together for the convenience of observers, and contains only such corrections as materially alter the places or identification of nebulae found by W. Herschel. Full particulars about these and many other objects will be found in the notes added to the three catalogues in the recently published edition of Herschel's *Scientific Papers*.*

N.G.C.

- 171 to be struck out; III. 223 = 175.
867. The place of III. 2 is extremely uncertain.
937 may be = III. 198 (given under 1003).
1331 to be struck out. III. 959 = I.C. 324, $11^{\text{h}} 1^{\text{m}} 2^{\text{s}}$ of I. 60.
The place given by H. no doubt refers to I. 60.
1442. The identity with 1440 is certain.
1551 to be struck out, II. 464 = 1550.
1611 is probably = 1613.
1794 is = III. 268, R.A. $5^{\text{h}} 1^{\text{m}} 45^{\text{s}}$.
1961. The place of III. 747 is wrong, it is = I.C. 2133.
2139. The identity with I.C. 2154 is certain.
2167. IV. 44 cannot be = h. 378 but is probably a star of the 11th mag. 70° following it on the parallel.
2319 = VIII. 1 (B.) R.A. $6^{\text{h}} 52^{\text{m}} \pm$, P.D. anything between $87^{\circ} 48'$ and $86^{\circ} 33'$.
2332. II. 862. In the sweep it is $2^{\text{s}} p$, $2' n$ of II. 736. Is probably identical with one of Kobold's nebulae in the I.C.
2641. III. 983. Minute of R.A. is 32 (H. was right).
2677. Delete "II. 48?"
2704. III. 625 is probably = I.C. 2424.
2722. III. 264. R.A. should be $8^{\text{h}} 51^{\text{m}} 46^{\text{s}}$.
2774. III. 61 is a different object, a S irr. cl., $56'$ south.
2938 is not III. 963 (only a star on Greenwich plate). The place of III. 963 is $9^{\text{h}} 23^{\text{m}} 17^{\text{s}}$, $13^{\circ} 3' 6''$.
2977. I. 282. The place according to Greenwich plate is $9^{\text{h}} 29^{\text{m}} 42^{\text{s}}$, $14^{\circ} 30' 5''$.
3061. The place of II. 903 (Greenwich) is $9^{\text{h}} 42^{\text{m}} 10^{\text{s}}$, $13^{\circ} 28' 7''$. Nothing in the place of h. 653 (h. says "very doubtful, moon and haze").
3063 is II. 333.
3065 is II. 334.
3066 is II. 909.

* A very few of the errors might have been found without the aid of the manuscripts. They had previously escaped my attention, because I assumed that Sir John Herschel had compared his positions throughout with those found by Auwers from W. Herschel's published observations (compare *Phil. Trans.*, 1864, pp. 4, 5). It appears now that this comparison cannot have been an exhaustive one.

N.G.C.

- 3110 is = II. 305.
 3122 to be struck out (= 3110).
 3144 is = III. 964.
 3155 is = III. 965.
 3174 to be struck out (= 3144).
 3183 is = I. 283.
 3194 to be struck out (= 3155).
 3197. III. 966. Greenwich place is $10^{\text{h}} 16^{\text{m}} 16^{\text{s}}$, $11^{\circ} 29' 6''$.
 3210. Minute of R.A. is 13.
 3218 to be struck out (= 3183).
 3284. III. 912 is no doubt identical either with III. 917 or 918,
 as H. did not see more than two nebulae here.
 3329 is = I. 284.
 3342. The place of III. 5 is extremely doubtful.
 3345. I. 26 is probably M. 95, as the latter was not mentioned
 by H.
 3381. Delete "1st of 3," which words belong to 3379.
 3397. To be struck out (= 3329).
 3401. III. 88 only seen once, $1^{\text{m}} 9^{\text{p}}$, $3^{\text{r}} n$ of II. 131.
 3465 is = III. 967.
 3484 h. 802. No trace of this on Greenwich plate.
 3497. To be struck out, III. 824 is = 3528 (error of reduction
 of 6^{m}).
 3500. III. 967 is = 3465, place of III. 968 is $10^{\text{h}} 51^{\text{m}} 23^{\text{s}}$, $14^{\circ} 2' 9''$.
 3523. II. 904. Place is $10^{\text{h}} 52^{\text{m}} 43^{\text{s}}$, $14^{\circ} 8' 0''$ (Greenwich).
 3528 is III. 824.
 3577. III. 723 observed only once, while there are two observations
 of II. 728.
 3604. II. 626. Only one observation in thick fog.
 3632. II. 30 is = 3626.
 3747. The place of III. 969 is $11^{\text{h}} 23^{\text{m}} 46^{\text{s}}$, $14^{\circ} 51' 0''$ (Greenwich
 plate).
 3752. The place of II. 905 is $11^{\text{h}} 23^{\text{m}} 46^{\text{s}}$, $14^{\circ} 36' 0''$ (Greenwich
 plate; h. 917 was eF on the plate).
 3794. To be struck out (= 3804).
 3804. II. 830 and III. 773 are identical.
 3837 is = III. 377.
 3842 is = III. 378.
 3851 is not = III. 378.
 3852. III. 36, the R.A. is possibly 1^{m} too great (not found by
 Bigourdan).
 3890. III. 940 = III. 971. Minute of R.A. is 41, not 40.
 3901. Place of III. 970 is $11^{\text{h}} 34^{\text{m}} 19^{\text{s}}$, $11^{\circ} 51' 0''$ (Greenwich).
 3922. III. 716 is = II. 825.
 3924 to be struck out (= 3922).
 3987 = III. 323.
 3993 = III. 324.
 4005 = III. 325.
 4007 to be struck out (= 4005, P.D. $64^{\circ} 6'$).

N.G.C.

- 4099 is probably to be struck out. H saw three, probably 4093, 95, 98.
- 4208 to be struck out (= 4212).
4209. Place perhaps doubtful.
- 4212 is also = II. 107.
- 4223 to be struck out (= 4241).
- 4241 is = II. 137, while III. 480 = I.C. 3115.
- 4263 III. 535 = 4265.
4277. For II. 571 read II. 570.
4281. For II. 570 read II. 571.
- 4341, 42, 43 = III. 94, 95, 96. All three observed by Kobold.
- 4362 } III. 800-801. Very probably the word "two" in H.'s
4364 } description refers to III. 799 and 800, as nobody seems
to have seen three nebulae about this place.
4368. III. 38 R.A. possibly 1^m too great (not found by Bigourdan).
- 4381 probably = 4357.
- 4472 is also = I. 7.
- 4520 is III. 757, not II. 757.
4588. III. 98 may be either I.C. 3591 or I.C. 3617, as H. did not observe the P.D. Nothing on Wolf's plate in the place of 4588.
- 4610 to be struck out; II. 19 = II. 498.
- 4646 has been numbered II. 910.
- 4664 and 4665 are identical.
- 4669 is = III. 778.
- 4675 is = II. 795.
- 4686 is = II. 796.
- 4695 is = III. 985.
4794. For II. 538 read III. 538.
- 4894 is not = III. 363, the latter is I.C. 4051.
- 4899 in the description, for eL read cL.
- 4954 is = III. 937.
4962. Identity of III. 303 is very doubtful.
- 4972 to be struck out (= 4954).
4979. III. 346 must be = I.C. 4198, as Wolf's ninth list has only one object there (No. 105).
5106. II. 22 must be = 5100.
- 5113 to be struck out; III. 808 = II. 826 (5109), each observed only once and in different sweeps.
5201. II. 797. Minute of R.A. is 23, not 24.
5255. III. 803. The R.A. given is the mean of two observations differing 1^m .
5396. III. 125, probably = 5375.
5484. III. 791 should probably be struck out. There is not any nebula close p I. 232.
5594. III. 135. The P.D. should be $63^\circ 8'$.
5640. III. 949. A star nearer to it in the sweep gives R.A. = $14^h 25^m 17^s$.
5699. To be struck out, is = 5706.

N.G.C.

5703. To be struck out, is = 5709.

5706 is = III. 127.

5709 is = III. 128.

5836. III. 312. R.A. is $15^h 0^m 5^s$ (Bigourdan).

5865 to be struck out; II. 684 = 5868.

5881. II. 818. Minute of R.A. is 4, not 6. Is probably = I.C.

1100.

5897 is not = VI. 8. The latter cannot be identified with certainty.

6064. III. 140. The identity with 6052 is certain.

6125. II. 810 is no doubt = 6127.

6526. Degree of P.D. of V. 9 is 114, and it is = I.C. 1271.

A Suggested Application of Mr. R. T. A. Innes' Formula for the Magnitude of Double Stars to the Observation of certain Variable Stars. By M. E. J. Gheury.

If A be the principal star of a pair A B sufficiently close to appear as a single source of light C, if a, b, c are the numerical magnitudes of A, B, C and L_A, L_B, L_C are their luminous intensities respectively, then, if $b - a = d$ (1) and $a - c = x$ (2), one has

$$L_B = \frac{L_A}{2 \cdot 512^d}, \quad \text{and} \quad L_C = L_A \times 2 \cdot 512^x.$$

As $L_C = L_A + L_B$, this gives

$$L_A \times 2 \cdot 512^x = L_A \left(1 + \frac{1}{2 \cdot 512^d} \right),$$

hence

$$x = \frac{\log \left(1 + \frac{1}{2 \cdot 512^d} \right)}{0 \cdot 4},$$

which is the formula proposed by Mr. R. T. A. Innes for the correction to apply to the magnitude of the principal star of the pair to obtain the "combined magnitude" (Reference Catalogue of Southern Double Stars, Annals of the Royal Observatory, Cape of Good Hope).

Mr. Innes gives a table of computed values of x to facilitate the use of the above formula, and shows its application to two different cases.

Although, as a rule, if one component of a binary be variable the other is also subject to variations of brightness, it is suggested that this formula may become particularly useful in the case of a variable star A having a close companion B of known constant magnitude. The observation gives in this case the value of c , and b is known, so that, since $c = a - x$ and $a = b - d$, $b - c = (d + x)$.

Mr. Innes' table is not adapted to this case, which he did not consider. The following modified and extended table is proposed, as it is believed to be more convenient, and its range of usefulness is greater.

d	x	$d+x$
0.0	0.8	0.8
0.1 and 0.2	0.7	0.8 and 0.9
0.3 „ 0.4	0.6	0.9 „ 1.0
0.5 to 0.7	0.5	1.0 to 1.2
0.8 „ 1.0	0.4	1.2 „ 1.4
1.1 „ 1.4	0.3	1.4 „ 1.7
1.5 „ 2.1	0.2	1.7 „ 2.3
2.2 „ 3.3	0.1	2.3 „ 3.4
3.4 and above	0.0	3.4 and above

For instance, if the companion is 6.4 mag. and the observation gives 4.8 for the combined magnitude, $b - c = 1.6 = d + x$, and, from the table, this is to be split into $x = 0.3$ and $d = 1.3$, giving $\alpha = 5.1$.

*Photographs of the Partial Solar Eclipse of 1912 April 17,
taken at the Royal Observatory, Greenwich.*

(Communicated by the Astronomer Royal.)

The solar eclipse of 1912 April 17 being one of unusually large magnitude, as seen at Greenwich, it was arranged to obtain a series of photographs of it with the Thompson photoheliograph of 9 inches aperture, in order to determine the relative positions of the Sun and Moon, and hence a correction to the Moon's place. The day proved fine, and a series of fifteen photographs was secured at the times given below. The exposing shutter of the photoheliograph was made to give in its fall a contact with wires connected with the chronograph, so that the times at which the exposures were made were automatically registered.

The photographs were measured by means of the position-micrometer in constant use for the measurement of the areas and positions of the spots and faculæ on the solar photographs obtained day by day at Greenwich, the Cape, and in India. This micrometer gives polar co-ordinates, and is well adapted for obtaining the centre of a circular object and of making measures of distance and position-angle from that centre. But as it is intended for the measurement of the solar photographs of daily routine, and as these are necessarily taken with their centres as nearly as possible coincident with the centre of the plate, no arrangement has been provided for centring on an object considerably removed from the centre of the plate, and only five of

the fifteen photographs could be measured in more than one position—viz. that in which the micrometer was centred on the

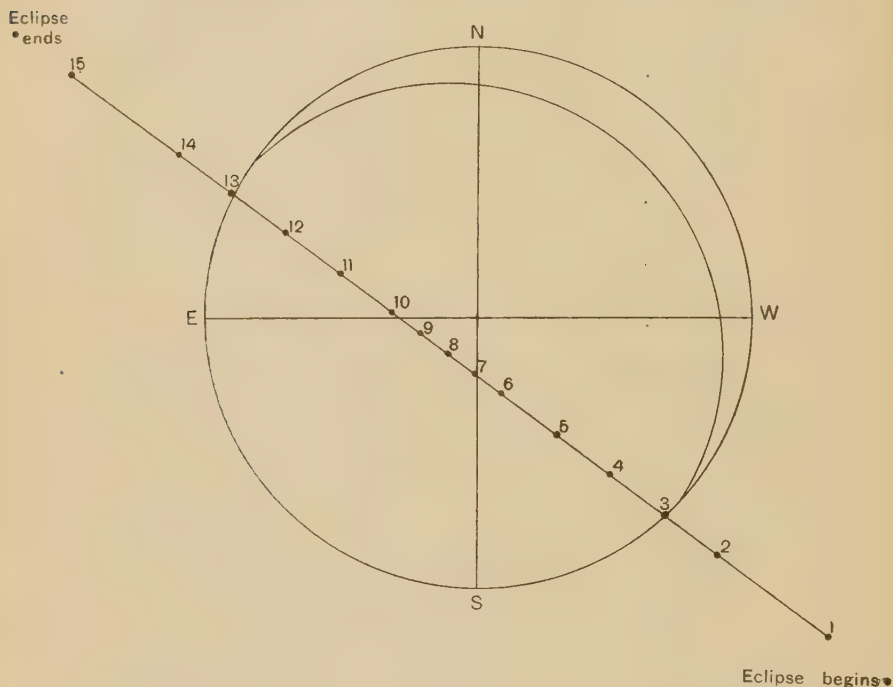


Diagram showing the positions relative to the Sun of the centre of the Moon at the times when the fifteen photographs were severally taken, and of the outline of the Moon at the time of the eighth photograph.

centre of the Sun. The five photographs showing the greatest phases of the eclipse were, however, also centred on the centre of the Moon.

TABLE I.

No. of Plate.	Greenwich Mean Time of Exposure.					No. of Plate.	Greenwich Mean Time of Exposure.				
	d	h	m	s			d	h	m	s	
1	1912 April 16	23	0	43	29	9	1912 April 17	0	15	30	89
2			20	39	81	10			20	29	92
3			30	38	05	11			30	28	22
4			40	36	62	12			40	26	62
5			50	34	96	13			50	25	03
6	17	0	0	33	52	14		1	0	23	31
7			5	32	52	15			20	18	03
8			10	31	50						

The photographs were measured in the following manner:— The plate was placed in the micrometer and moved by the proper adjusting screws until, on rotating the position-circle, the cross-lines in the centre of the field of view of the magnifier—which was firmly clamped—remained on the limb of the Sun in all position-angles. In centring on the Moon the same procedure was adopted, except that an average position of the Moon's limb had to be taken, the irregularities in its contour being considerable.

When the plate had been thus centred on the Sun the magnifier was fixed in distance at some convenient reading, and the micrometer rotated until first one point on the Moon's limb and then another were brought under the cross-lines, the position-circle being read at each setting. In this way a large number of pairs of settings were obtained on the limb of the Moon, extending from either cusp to near the point of nearest approach of the limb of the Moon to the centre of the Sun; that is to say, to the line joining the centres of the two bodies, the direction of that line being given by the mean position-angle derived from each pair of position-circle readings. Then a few readings were taken on that line, and at short distances on either side of it, the position-circle being clamped at the desired reading and the magnifier moved in distance until the cross-lines were on the limb of the Moon. An analogous procedure was adopted in the measurements made with the micrometer centred on the Moon.

The following method was employed in the reduction of the photographs:—

If P be a point on the circumference of the Moon and M the centre of the Moon, while S is supposed to be at once the centre of the Sun and the centre of rotation of the measuring instrument, then in the triangle PMS

$$PS = PM \cos P + SM \cos S$$

or

$$r = a \cos \phi - c \cos \theta$$

where

$r = PS$, the distance of P from the centre of the Sun,

$a = PM$, the radius of the Moon,

$c = SM$, the distance between the two centres,

$\phi = P$, the angle at P between the directions of the two centres,

$\theta = 180^\circ - S$, S being the difference in position-angle between P and the Moon's centre.

If ψ be the observed position-circle reading of P and δ that of the point where the line through the centres meets the Moon's limb, then $\theta = \psi - \delta$.

Adopting from the inspection of the measures approximate values for a , c , and δ , the distance, r , corresponding to each of the several position-circle readings was computed from the formula:—

$$\begin{aligned} r &= a \cos \phi - c \cos \theta, \\ a \sin \phi &= c \sin \theta, \end{aligned}$$

where

and corrections to the assumed values of a , c , and α were then computed from the formula

$$\Delta r = \sec \phi \{ \Delta a - \Delta c \cos (\theta - \phi) - c \sin (\theta - \phi) \Delta \delta \}.$$

The five photographs that were also measured with the centring made upon the Moon were reduced in an analogous manner.

As the measures were made in pairs symmetrically placed with respect to the line joining the centres, a series of equations was formed also in pairs in which the coefficients of Δa and Δc remained the same, but those of $\Delta \delta$ were equal in amount but opposite in sign, so that by their combination either $\Delta \delta$, or Δa and Δc together, could be eliminated. A least-square determination of Δa was determined from the equations involving Δa and Δc for each of the five photographs of greatest phase—Nos. 6 to 10,—and the mean of the five resulting values of the lunar radius was adopted, after correction for the effect of parallax, in the solution of the equations for the determination of Δc . The following table gives the position-angles of the centre of the Moon from the centre of the Sun, and the distances, expressed in scale-reading and in seconds of arc, between the centres.

TABLE II.

Photographs centred on the Sun.

No. of Plate.	Position-Angle of Moon's Centre.	Distance of Centres.	
		Scale.	Arc.
		d	
1	227° 25'·2	1698'·47	1659''·41
2	225° 30'·8	1213'·53	1185'·62
3	223° 29'·3	971'·44	949'·10
4	220° 52'·5	735'·78	718'·86
5	214° 59'·0	500'·05	480'·55
6	199° 13'·9	279'·10	272'·68
7	178° 32'·9	189'·63	185'·27
8	139° 6'·2	156'·39	152'·79
9	102° 46'·0	202'·63	197'·97
10	84° 30'·2	297'·40	290'·56
11	70° 25'·8	520'·59	508'·62
12	65° 5'·2	751'·29	734'·01
13	62° 21'·2	989'·20	966'·45
14	60° 34'·7	1226'·31	1198'·10
15	58° 54'·1	1708'·24	1668'·95

Photographs centred on the Moon.

No. of Plate.	Position-Angle of Sun's Centre.	Distance of Centres.	
		Scale.	Arc.
		d	
6	19° 10'·7	279'·30	272''·88
7	358° 35'·5	190'·70	186'·31
8	319° 16'·3	154'·62	151'·06
9	282° 1'·2	203'·51	198'·83
10	264° 22'·8	297'·93	291'·08

The vernier of the scale of the position-micrometer reads approximately to the tenth of a millimetre, one division of the vernier being nearly equivalent to a second of arc; the value of a division was determined from the first and last photographs—the amount of the motion of the Moon in the interval being accurately known—to be

$$1^d = 0''.977.$$

Table III. gives the apparent errors of the tabular place of the Moon as shown by each photograph. The tabular distances between the centres of the Sun and Moon, position-angles of the Moon's centre, and differences of R.A. (measured on a great circle) and of Decl. between the Sun and Moon, were supplied for the times of the several photographs by Dr. A. C. D. Crommelin.

TABLE III.

Photographs centred on the Sun.

No of Plate.	Tabular Distance of Centres.	Tabular Position- Angle.	Difference of R.A. Sun-Moon (Great Circle).		App. Error Tab. R.A. of Moon (Great Circle).	Difference of Decl. Sun-Moon.		App. Error Tab. Decl. of Moon.
			Tabular.	Measured.		Tabular.	Measured.	
1	1671''54	227° 30' 20	+ 1232''46	+ 1221''88	- 10''58	+ 1129''21	+ 1122''79	- 6''42
2	1194'90	225 32'38	+ 852'86	+ 845'84	- 7'02	+ 836'93	+ 830'82	- 6'11
3	959'03	223 49'42	+ 664'07	+ 653'18	- 10'89	+ 691'91	+ 688'59	- 3'32
4	725'19	220 59'52	+ 475'69	+ 470'43	- 5'26	+ 547'37	+ 543'56	- 3'81
5	495'64	215 28'30	+ 287'62	+ 280'10	- 7'52	+ 403'66	+ 400'28	- 3'38
6	279'02	200 54'89	+ 99'60	+ 89'82	- 9'78	+ 260'63	+ 257'46	- 3'17
7	189'56	181 43'27	+ 5'69	- 4'69	- 10'38	+ 189'47	+ 185'21	- 4'26
8	147'76	143 17'24	- 88'33	- 100'03	- 11'70	+ 118'44	+ 115'49	- 2'95
9	188'55	104 36'46	- 182'45	- 193'08	- 10'63	+ 47'55	+ 43'75	- 3'80
10	277'52	85 14'27	- 276'57	- 289'22	- 12'65	- 23'04	- 27'83	- 4'79
11	493'21	70 36'05	- 465'20	- 479'24	- 14'04	- 163'82	- 170'37	- 6'55
12	721'47	65 6'05	- 654'41	- 665'71	- 11'30	- 303'76	- 309'20	- 5'44
13	953'52	62 19'10	- 844'38	- 856'11	- 11'73	- 442'97	- 448'45	- 5'48
14	1187'33	60 40'57	- 1035'19	- 1043'58	- 8'39	- 581'49	- 588'55	- 7'06
15	1657'50	58 55'21	- 1419'55	- 1429'09	- 9'54	- 855'66	- 862'03	- 6'37

Photographs centred on the Moon.

6	279'02	200° 54' 89	+ 99'60	+ 89'64	- 9'96	+ 260'63	+ 257'74	- 2'89
7	189'56	181 43'27	+ 5'69	- 4'58	- 10'27	+ 189'47	+ 186'25	- 3'22
8	147'76	143 17'24	- 88'33	- 98'56	- 10'23	+ 118'44	+ 114'48	- 3'96
9	188'55	104 36'46	- 182'45	- 194'47	- 12'02	+ 47'55	+ 41'41	- 6'14
10	277'52	85 14'27	- 276'57	- 289'68	- 13'11	- 23'04	- 28'51	- 5'47
Mean of the 20 determinations					- 10'3			- 4'735

The form of the position-micrometer was well adapted for the measurement of the eclipse negatives up to a certain point, the determinations of the direction of the line of centres being given with precision by the method of fixing the magnifier in distance, and taking the readings of the position-circle where the cross-lines met the circumference of the Moon. The successive pairs of readings thus obtained gave values for the position-angle of the line of centres that were always very accordant.

But a great difficulty was experienced in determining the orientation of the plates, particularly those of greatest phase, as only a very small portion of two of the four wires could be seen. In the case of the photographs centred on the Moon the orientation could only be roughly estimated, and the differences in the final results obtained from these five photographs when centred on the Sun and when centred on the Moon are largely owing to this difficulty. It is probable that this difficulty would have been overcome to a considerable degree if the image had been moved before each exposure so as to bring the ends of three wires on the uneclipsed portion of the Sun. Or the wires might have been artificially illuminated, or else the field might have been lightly fogged, so as to show up the wires in their entire length.

As Table III. shows, and as was to have been expected, the photographs near the time of greatest phase give more accordant results than those when only a small portion of the Moon was projected against the Sun. It would seem worth while on another suitable occasion to multiply the photographs near mid-eclipse as much as possible, reserving those near the beginning and end mainly for the purpose of determining the scale value.

The photographs were all rather lightly exposed, the same plates and development being used as for the daily record of the Sun's surface. A fuller exposure, and development calculated to give a harder, sharper image, would probably have given negatives more suitable for measurements of the kind required, but the chief result would have probably been merely to give a somewhat larger radius to the Sun, and a smaller one to the Moon. The photographs actually secured gave nearly the same radius to the two bodies, the mean radius from the five central photographs being for the Moon—

$$15' \ 54''.99,$$

and for the Sun—

$$15' \ 55''.26.$$

The exposures of the photographs, their measurement, and the reduction of the measures, were made by Mr. Maunder, assisted by Mr. A. H. Smith.

Royal Observatory, Greenwich :
1912 Nov. 7.

*Mean Areas and Heliographic Latitudes of Sun-spots
in the Year 1911.*

(Communicated by the Astronomer Royal.)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lxxii. p. 3, and are deduced from the measurements of photographs taken at the Royal Observatories of Greenwich and of the Cape, and in India at the Kodaikānal Observatory and at Dehra Dûn, North-West Provinces.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in the year 1911, and Table II. gives the same particulars for each of the entire years 1901 to 1911 inclusive. The areas are given in two forms: first, projected areas, that is to say, as seen and measured on the photographs, these being expressed as millionths of the Sun's apparent disc; and next, areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in the year 1911 the mean daily area of the whole spots (corrected for foreshortening), and the mean heliographic latitude of the spotted areas for spots north and for spots south of the equator, together with the mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots; and Table IV. gives the same information for each year as a whole for the years 1901 to 1911 inclusive. Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*, and for the years 1889 to 1902 on pp. 465 and 466 of vol. lxiii., and for the years 1901 to 1910 on pp. 4 and 5 of vol. lxxii.

TABLE I.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Mean of Daily Areas.					
			Projected.			Corrected for Fore-shortening.		
			Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
	1910. d							
766	Dec. 27'53	26	14	61	577	11	45	579
	1911.							
767	Jan. 23'86	27	18	91	539	11	61	532
768	Feb. 20'21	28	20	119	652	16	93	667
769	Mar. 19'53	27	42	226	697	32	179	686
770	Apr. 15'81	27	50	268	735	36	194	712
771	May 13'06	27	17	110	452	13	78	472
772	June 9'26	28	1	10	338	1	6	343
773	July 6'46	27	3	18	393	2	10	394
774	Aug. 2'67	27	10	47	476	7	31	485
775	Aug. 29'91	27	16	88	372	11	59	379
776	Sept. 26'17	28	8	33	253	5	20	258
777	Oct. 23'46	27	2	11	257	1	6	259
778	Nov. 19'77	27	16	88	285	11	63	299
779	Dec. 17'08	28	4	19	246	3	12	259

TABLE II.

Year.	No. of Days on which Photo- graphs were taken.	Mean of Daily Areas					
		Projected.			Corrected for Foreshortening.		
		Umbrae.	Whole Spots.	Faculae.	Umbrae.	Whole Spots.	Faculae.
1901	359	14	41	23	9	29	29
1902	349	14	86	163	10	62	178
1903	350	67	434	875	51	340	970
1904	363	93	653	1639	67	488	1761
1905	364	230	1637	2433	163	1191	2612
1906	364	158	1047	2207	114	778	2320
1907	365	221	1453	1859	160	1082	1999
1908	366	148	952	2000	106	697	2098
1909	364	171	941	1303	124	692	1353
1910	365	67	357	984	49	264	971
1911	364	16	88	458	12	64	459

TABLE III.

No. of Rota- tion.	Date of Com- mencement of each Rotation.	No. of Days on which Photo- graphs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heli- graphic Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			Mean of Daily Areas.	Mean Heli- graphic Lati- tude.	Mean of Daily Areas.	Mean Heli- graphic Lati- tude.		
766	1910. d Dec. 27'53	26	0	...	45	4°63	- 4°63	4°63
767	1911. Jan. 23'86	27	0	...	61	12°26	- 12°26	12°26
768	Feb. 20'21	28	2	2°85	91	4°19	- 4°06	4°17
769	Mar. 19'53	27	51	9°81	128	5°55	- 1°15	6°77
770	Apr. 15'81	27	0	...	194	5°65	- 5°65	5°65
771	May 13'06	27	73	8°55	5	6°78	+ 7°54	8°43
772	June 9'26	28	4	2°11	3	11°08	- 3°27	5°77
773	July 6'46	27	9	6°48	1	12°12	+ 4°24	7°16
774	Aug. 2'67	27	6	6°13	25	11°90	- 8°40	10°78
775	29'91	27	58	6°47	1	4°59	+ 6°27	6°44
776	Sept. 26'17	28	20	6°81	0	...	+ 6°81	6°81
777	Oct. 23'46	27	0'4	1°70	5'5	3°69	- 3°32	3°55
778	Nov. 19'77	27	0	...	63	2°08	- 2°08	2°08
779	Dec. 17'08	28	5	10°44	7	14°44	- 3°54	12°69

TABLE IV.

Year.	No. of Days on which Photo- graphs were taken.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Heli- ographic Latitude of entire Spotted Area.	Mean Dis- tance from Equator of all Spots.
		Mean of Daily Areas.	Mean Heli- graphic Lati- tude.	Mean of Daily Areas.	Mean Heli- graphic Lati- tude.		
1901	359	22	8°59	7	16°27	+ 2°82	10°37
1902	349	42	18°81	21	15°29	+ 7°48	17°64
1903	350	132	18°11	208	21°15	- 5°85	19°94
1904	363	268	16°33	220	16°88	+ 1°37	16°57
1905	364	750	11°66	440	15°55	+ 1°60	13°10
1906	364	539	13°98	239	14°01	+ 5°38	13°99
1907	365	488	10°12	593	13°77	- 2°98	12°12
1908	366	316	10°42	381	10°34	- 0°92	10°38
1909	364	299	9°45	393	9°92	- 1°55	9°71
1910	365	66	8°55	198	11°18	- 6°27	10°53
1911	364	17	7°89	47	5°99	- 2°30	6°49

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

The principal features of the record for 1911 are:—

1. The continuance of the rapid decline in the mean daily spotted area that began in 1910. Thus in 1911 the mean daily area only reached 24 per cent. of its value in 1910, or 9 per cent. of that for 1908 or 1909. Comparing the mean area for 1911—viz. 64 millionths of the Sun's visible hemisphere—with that recorded for the year immediately preceding the minimum in the last three cycles, it would appear that the actual minimum may be expected to fall in the current year, and that the beginning of the revival may be looked for in 1913.

Year before Minimum.	Mean Daily Area.	Year of Minimum.	Mean Daily Area.
1877	108	1878	22
1888	89	1889	78
1900	75	1901	29

2. An examination of the results for the separate rotations shows that the year 1911 maintained much the same low level of activity throughout, except for a feeble revival in the month of April which was followed by a strongly marked minimum in June and July.

3. The curve shown by the areas of the faculæ has been, as usual, much smother than that for the spots, though the revival of April was apparent in it also. The decline in area, also, is less marked than with the spots, the figure for 1911—459—being 47 per cent. of that for 1910 instead of only 24 per cent. Comparing this area with those obtained in the preceding cycles, the year most nearly corresponding in faculous area to the record of 1911 falls earlier in the cycle than is the case when the spotted area is considered.

Year.	Mean Daily Area.	Year of Minimum.	Mean Daily Area.
1875	503	1878	67
1887	301	1889	131
1899	337	1901	29

The state of the faculæ would, therefore, rather indicate that the minimum would fall in 1913 or even in the year following.

4. Comparing the two hemispheres, the southern, as in 1910, has been about three times as prolific as the northern; indeed the northern hemisphere has been completely free from spots in four rotations out of fourteen, and very nearly free in about six others. The southern hemisphere has been completely free from spots in one rotation, and nearly so in six. The preponderance of the southern hemisphere, characteristic of the period of decline, has been nearly as marked in 1911 as in 1910.

5. The mean distance from the equator of all spots has shown a great falling-off as compared with the three previous years, being $6^{\circ}49'$ in 1911 as compared with $10^{\circ}53'$ in 1910.

6. The Sun was free from spots on 183 days in 1911, as compared with 65 in 1910.

7. In the distribution of spots in latitude, the disturbances in the southern hemisphere have ranged from the equator to 16° , and in the northern to 11° . There have also been very feeble disturbances in each hemisphere in comparatively high latitudes; viz. 20° in the southern and 24° in the northern hemisphere.

8. The number of separate groups observed in 1911 was 62 instead of 151 as in 1910; or, expressed as a percentage of that year, it would stand as 41. This is a smaller decline than that in area, and shows that the proportion of small and short-lived groups continues to increase.

9. Of the 62 separate groups of the year, 42 were in the southern and 20 in the northern; or, expressed in percentages, 32 north to 68 south; very nearly the same proportion as in 1910. The southern hemisphere has now shown the larger number of groups for five years running.

Royal Observatory, Greenwich :
1912 November 6.

The whole region of the chromosphere between 228° and 250° was raised above the normal. The prominence at P.A. 250° was also seen on October 9, height $26''$. Although there were not any prominences above the medium height, yet the western limb of the Sun was much more active than the eastern limb. In addition the sun-spot group and its accompanying faculæ were approaching the same limb. We might therefore expect to find the Sun's corona more diversified on the Sun's west limb.

At my request Mr Buss of Manchester made observations of the disc phenomena and prominences on the day of the eclipse, and permits me to quote from his observations. The instruments employed were a Rowland grating 14,438 lines to the inch, ruled surface $1\frac{3}{4} \times 1\frac{1}{4}$, attached to a 3-inch equatorial. The first order of the spectrum is employed for prominences, the second order for disc phenomena. The observations were made October 10, $1^h 20^m$ to $1^h 45^m$ G.M.T. Over the area of faculæ surrounding the spot-group dark D_3 was observed in broken portions of short lines. The widened slit showed bright prominences in projection, and dark H_{α} flocculi. The limb of the Sun was devoid of conspicuous prominences, but displayed great roughness of chromosphere in many places and was marked by faculous regions and small bright spikes. This was more conspicuous in the N.E. and S.E. quadrants, and at P.A. 110° there was an apparent raising of the chromosphere into a long flat ridge. The characteristics of spikiness and roughness were also marked from 235° to 270° . At 245° were some radially set and mostly detached filaments and streaks, at 255° a very rough and spiky portion of the chromosphere and a dull south-slanting prominence of indefinite outline. Then followed a rough chromosphere to 335° , and from this reading to 345° , a group of three or four small prominences, not very bright.

Stonyhurst College Observatory :
1912 October 25.

Sun-spots and Terrestrial Magnetic Phenomena, 1898-1911: the Cause of the Annual Variation in Magnetic Disturbances. By the Rev. A. L. Cortie, S.J.

1. *Sun-spots and Terrestrial Magnetic Phenomena, 1898-1911.*

The general accord between solar spots and magnetic storms has been established from the Stonyhurst records 1881-1898 by Father Sidgreaves (*Memoirs R.A.S.*, liv. pp. 85-96). Similar data have been employed in constructing the following table and the accompanying curves, which set forth a comparison between the mean daily disc-area of sun-spots, the daily range of the horizontal direction and force, and the numbers of magnetic storms, very great, great, and moderate, for the period 1898-1911, which covers a complete solar cycle. The disc-areas of the spots are expressed

in $\frac{1}{5000}$ of the area of the visible disc of the Sun. The declination ranges are in minutes of arc, and the horizontal force ranges in 10^{-5} C.G.S. unit of force.

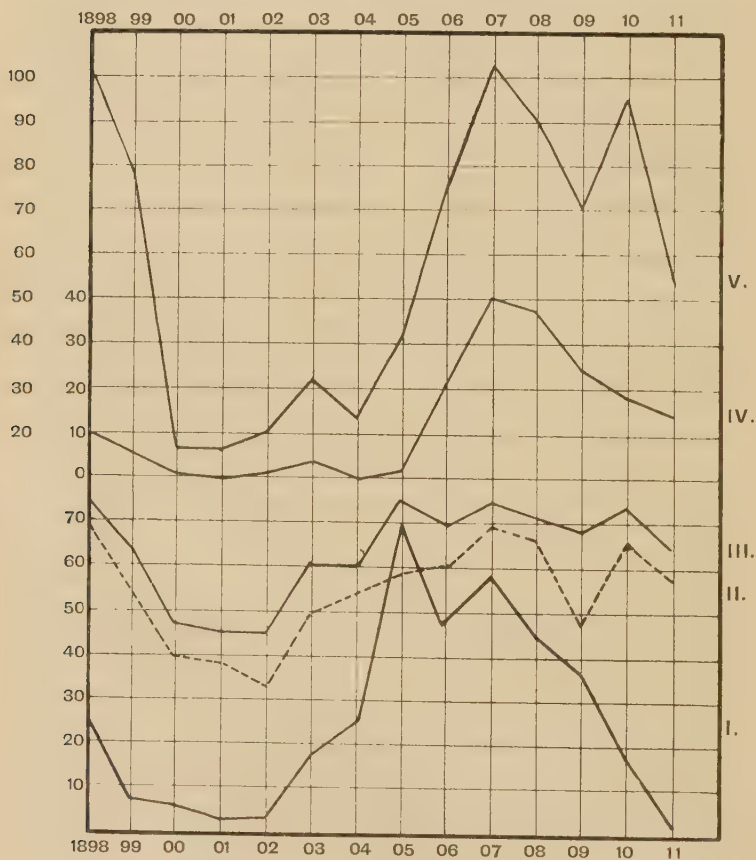
TABLE I.

Year.	Mean Daily.			Number of Magnetic Storms.		
	Disc-area Sun-spots.	Horizontal Range.		Very Great.	Great.	Moderate.
		Direction.	Force.			
			10-5			
1898	2.50	14.7	69	2	8	102
1899	0.74	12.9	55	0	7	79
1900	0.55	9.7	41	0	2	16
1901	0.29	9.1	39	0	0	16
1902	0.33	9.0	34	0	1	20
1903	1.93	11.8	50	2	2	32
1904	2.54	11.9	48	0	0	24
1905	6.88	14.9	59	0	2	41
1906	4.79	13.9	60	1	20	75
1907	5.75	14.7	69	2	38	103
1908	4.55	14.1	67	4	34	91
1909	3.76	13.5	57	5	19	71
1910	1.80	14.5	66	0	19	95
1911	0.33	12.7	59	0	15	56

We have two salient features to account for—a retardation of the maxima of magnetic force and storms on the sun-spot maximum, and the inversion of magnetic activity and sun-spot area in 1909-10.

On 1909 September 25 occurred a magnetic storm of exceptional violence. The suddenness of the commencement of such storms, and the practical simultaneity of such commencement, at magnetic stations scattered all over the earth, indicate that at least their instrumental, if not their efficient cause, is something extra-terrestrial. This storm was accompanied by six other great magnetic storms in the September and October of the same year (*M.N., R.A.S.*, lxx. pp. 19-23). The recovery of magnetic activity in 1909-10 was heralded by these great storms. All the greater sun-spots of September and October 1909 were confined within a belt of about 10° each side of the Sun's equator. The mean latitude of all spots for 1909 was N. 9.5° S. 9.9° ; for 1910, N. 8.6° S. 11.2° , the total spotted area having greatly diminished. We must also take account of the general distribution in latitude of sun-spots, which is thus enunciated by Mr. Maunder (*M.N., R.A.S.*, lxiv, p. 756): "During the rise to maximum the spots avoid the equatorial regions, but after maximum they gradually abandon the higher latitudes and close

in towards the equator." The maximum heliographic latitude of the Earth is $\pm 7^{\circ} 15'$, and therefore it would only come into a position of advantage relatively to the spot zones, after the maximum of spot activity had passed. Hence is explained the



CURVES, 1898-1911.

I. Sun-spots Mean Daily Disc-area. II. Mean Daily Range Horizontal Magnetic Force. III. Magnetic Direction. IV. Yearly Numbers Great Magnetic Storms. V. Moderate Magnetic Disturbances. The numbers for declination ranges are multiplied by 5.

retardation of greatest magnetic upon greatest spot activity. It was not until 1907 that the mean latitudes of sun-spots approached the zone of 10° . Again, the spots, reduced in area, were still active in 1909 and the beginning of 1910, with mean latitudes not far removed from the Earth's maximum heliographic latitude. The position of advantage relatively to the Earth was still

maintained and will equally well explain the inversion of sun-spot area and magnetic activity.

The relation between solar prominences and terrestrial magnetism has already been discussed (*Ap. J.*, xviii. pp. 287-293), with negative results. In the present cycle of solar activity the maximum of profile-area from the Kodaikānal observations was reached in 1908, after the maxima of magnetic storms and force, and the maximum of mean daily frequency in 1910, due to an increase in small prominences as the minimum of solar activity was approached. The mean yearly latitudes of the prominences seem also to be too high to affect the Earth's magnetism. The connection between prominences and magnetic activity is very problematical.

2. *The Cause of the Annual Variation in Magnetic Disturbances.*

In Dr. Chree's recent work, *Studies in Terrestrial Magnetism* (pp. 189, 190), he gives the following table compiled from all the magnetic storms in Mr. Maunder's tables:—

Annual Distribution of Storms at Greenwich.

		Winter.	Equinox.	Summer.
Sun-spot maximum	. .	35	38	27
„ minimum	. .	28	48	24

The figures express the number for each season as a percentage of the total of the year. Dr. Chree thus comments on these numbers: "Taking the fifty-six years covered by Mr. Maunder's investigation, equinox possessed considerably more than its fair share—one-third—of the disturbances. The table shows that this peculiarity of the equinoctial months is much more prominent in years of few than in years of many sun-spots. It would appear that at Greenwich as sun-spots increase the incidence of prominent magnetic disturbance tends to be more uniform, alike throughout the twenty-four hours and the twelve months." This subject of the seasonable variation in magnetic disturbances was discussed by Mr. Ellis for the fifty years 1848-97, in *Monthly Notices R.A.S.*, lxv. pp. 520-22. His curves and numbers showed that the points of maximum disturbance fall at about March 3 and October 9 respectively. He wrote: "This close agreement in the interval of time—in the one case preceding the equinox by some seventeen days and in the other following the equinox by some seventeen days—may be, in a sense, accidental; and it will be interesting to see how far this result may afterwards be confirmed." It was with this object that the following table and diagrams were prepared, the materials for which are contained in Father Sidgreaves' Annual Reports for the years 1889-1911. All personal bias therefore in the selection of magnetic disturbances, and in apportioning to each its order of magnitude, was eliminated. The heavy work,

too, of examining all these magnetic curves would have been beyond my possibilities of leisure for the investigation. The arrangement of the material at hand follows exactly the procedure adopted by Mr. Ellis, so that the results may be comparable. The smoothed values have been found by twice taking in succession the means of three observed values. The material used overlaps Mr. Ellis's period by ten years, and extends thirteen years beyond it.

TABLE II.

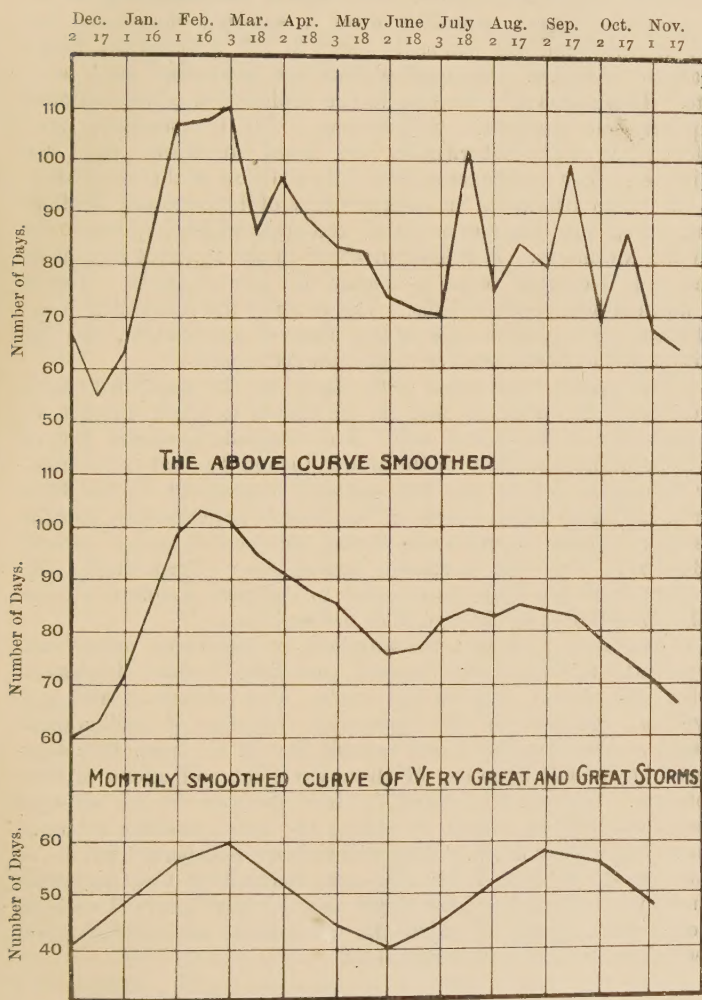
Number of Days of Magnetic Disturbance, 1889-1911, in Half-Monthly Periods.

First Day of Half- Monthly Period.	Total Number of Days of Disturbance		First Day of Half- Monthly Period.	Total Number of Days of Disturbance	
	As Observed.	As Smoothed.		As Observed.	As Smoothed.
Jan. 1	63	71	July 3	71	82
16	85	84	18	102	84
Feb. 1	107	98	Aug. 2	76	83
16	108	103	17	84	85
Mar. 3	110	101	Sept. 2	80	84
18	86	95	17	99	83
Apr. 2	97	91	Oct. 2	69	79
18	89	88	17	87	75
May 3	84	85	Nov. 1	68	71
18	83	80	17	64	67
June 2	74	76	Dec. 2	65	62
18	72	77	17	55	63

Inspection of the smoothed numbers and of the smoothed curve shows two maxima, one in the second half of February and the other in the second half of August, the February maximum being very much more pronounced than that in August. The minima are in the first half of June and the first half of December, the latter being the more pronounced. In fact the first half of the curve has a much greater amplitude than the second half.

The inclination of the Sun's axis to the plane of the ecliptic is $82^{\circ} 45'$. Hence the angle between the planes of the Sun's equator and the ecliptic is $7^{\circ} 15'$. The longitude of the ascending node of the Sun's equator is about 74° , and of the descending node 254° . The Earth therefore passes through the plane of the Sun's equator in the beginning of June and in the beginning of December. These positions of the Earth relatively to the plane of the Sun's equator correspond exactly to the minima of the smoothed curve. We may imagine the Earth's path as projected upon the Sun's disc, and consider the heliographic position of the Earth at different seasons of the year. The maxima of the above smoothed curve occur in the second half of February and the second half of August.

At these times the heliographic latitude of the Earth is approaching its maximum negative value and its maximum positive value. These values are absolutely attained at the beginning of March,



SEASONAL VARIATION OF MAGNETIC DISTURBANCE.

which corresponds to Mr. Ellis's first maximum, and at the beginning of September, his second maximum being at the beginning of October. But the correspondence of the Earth's maxima in heliographic latitude, and the maxima points of the curves in both discussions, is strikingly close.

We know that when sun-spots are diminishing in area and number they approach the equator of the Sun. But very few spots of any magnitude are actually on the equator of the Sun. Large and active spots do, however, occur within the regions $\pm 10^\circ$ of heliographic latitude, especially after a sun-spot maximum has been attained, and the sun-spots begin to fall towards the Sun's equatorial latitudes. Magnetic storms are connected with sun-spots. These facts give the clue to the explanation of the seasonal inequality in magnetic disturbances. All investigators have noticed that such disturbances are more numerous near the equinoxes. The equinoxes are not the condition of this inequality, but the Earth attains its maximum values of heliographic latitude when it gets into the region of the sun-spots which happen to be near the equinoxes. It is least involved in the sun-spot regions in June and December, when it crosses the Sun's equator. Hence the minima near those months. The cause of the inequality is the inclination of the Sun's axis to the plane of the equator, and the position of the Earth relative to the sun-spot zones.

It has already been noted with regard to the smoothed curve of magnetic disturbances that its amplitude is much greater in the first than in the second half. This suggests a crucial test of the hypothesis to account for the annual inequality, just enunciated. For it ought to follow, that the southern hemisphere of the Sun, corresponding to minus values of the Earth's heliographic latitude, was more prolific in sun-spots during the period under review, 1889-1911, than the northern hemisphere. Accordingly, the following table has been constructed to compare magnetic storms with sun-spot area in the two solar hemispheres.

Columns 2, 3, 4 give the numbers of moderate, great, and very great magnetic storms for each year from Father Sidgreaves' tables, and column 5 gives the totals. For columns 6 and 7 I have had recourse to the Greenwich volumes of heliographic results, and incidentally I may remark, that in all these investigations I have made large use of these excellent collections of sun-spot data. Column 8 is meant to show the flow of the sun-spot phenomena, and so instead of adding the two preceding columns, I have taken their mean. This avoids large numbers. For 1911 I have used the Stonyhurst disc-areas reduced to the Greenwich standard of millionths of the Sun's disc. I have marked with an asterisk those years in which the mean daily sun-spot area was greatest in the southern hemisphere.

TABLE III.

Comparison of the Numbers of Magnetic Storms and the Mean Daily Area of Sun-spots in the Sun's N. and S. Hemisphere for the Period 1889-1911.

Year.	Magnetic Storms.				Mean Daily Sun-spot Area.		Mean of the Means.
	M.	G.	V. G.	Total.	N.	S.	
*1889	125	23	0	148	5	73	39
1890	84	14	0	98	53	46	50
1891	119	12	0	131	401	169	285
1892	110	24	0	134	607	607	607
*1893	56	9	0	65	517	941	729
*1894	87	9	9	105	543	739	641
1895	107	8	0	115	565	409	487
*1896	82	4	4	90	203	340	272
*1897	92	5	0	97	196	318	257
*1898	102	8	2	112	110	266	188
*1899	79	7	0	86	23	88	56
*1900	16	0	0	16	26	49	38
1901	16	0	0	16	22	7	15
1902	20	1	0	21	42	21	32
*1903	32	2	2	36	132	208	170
1904	24	0	0	24	268	220	244
1905	41	2	0	43	750	440	595
1906	75	20	1	96	539	239	389
*1907	103	38	2	143	488	593	491
*1908	91	34	4	129	316	381	349
*1909	71	19	5	95	299	393	346
*1910	95	19	0	114	66	198	132
1911	56	15	0	71	...	...	33

Excluding 1911, for which the record of sun-spot area is not given for both hemispheres, we proceed to find the mean daily spot-area for each hemisphere for the remaining 22 years. It works out as 281 for the N. and 307 for the S. hemisphere. There is here a greater mean daily area for the S., but only 9 per cent. greater. But if the table be examined, it will be seen that in several years of greater sun-spot area there were fewer storms recorded than in years of much less sun-spot area. For instance, in 1910, a year approaching the minimum, there are more than twice as many storms as in 1905, a year of sun-spot maximum. Let us therefore count those years in which the preponderance of sun-spot area was greater in the N. and in the S. hemisphere respectively, excluding 1892, in which the areas were equal both N. and S., and 1911 for want of data. We thus have 21 years to

deal with. In 13 of these years the sun-spot area was greater in the S. and in 8 years greater in the N. hemisphere, a great preponderance for the S. hemisphere. Of magnetic storms 1236 were recorded in the years of greater S. sun-spot area, and 544 in the years in which sun-spot area was greater in the N. hemisphere. These figures work out at 95 per annum in the one case, and 68 per annum in the other, in the ratio of 1.4 to 1. It may be a purely accidental coincidence, but the ratio of the two maxima in the smoothed curve is nearly of the same order, 1.2 to 1. The result of this inquiry is, that magnetic storms in the period under discussion were more prolific in those years in which the mean daily area of spots was greater in the southern than in the northern hemisphere of the Sun. The theory proposed has stood the test which has been applied to it.

Again, great and moderate magnetic storms are very frequently the companions to, or the successors, in following synodic rotations of the Sun, of a very great magnetic storm. In *M.N. R.A.S.*, lxiv., pp. 208-209, Mr. Maunder gives a list of "great" magnetic storms, or what we should call for the most part "very great" on our system of nomenclature. I have prepared a similar list to that of Mr. Maunder for the years 1898 to 1912, three storms being common to the two lists. Taking these two lists together, and reckoning as one storm such storms as may have persisted for more than one day, the two lists give a total of 22 of these storms of the first order of magnitude. Of this number 5 only were connected with spots in the northern hemisphere, and the other 17, more than 3 to 1, with spots in the southern hemisphere. This result lends weight to the arguments already adduced in support of the explanation of the cause of the annual variation in magnetic storms. Finally, we may remark, that the same cause would be operative in the case of the annual variation of the daily ranges of the magnetic elements. Dr. Chree has established from his elaborate discussion of the Kew and Falmouth results that the semi-annual term in all cases has its maxima near the equinoxes, with minima near midwinter and midsummer; or, as we may express it, when the heliographic latitude of the Earth referred to the Sun's equator has its maximum and minimum values.

Stonyhurst College Observatory:
1912 November 1.